Enabling Mutualization and Prosumer Empowerment for Collective-Centric Optimization

Towards Responsible Energy Communities

by

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Foreword

Such a broad and complex topic as addressing electricity consumption and use may be tackled only by the efforts of a large scientific community. This thesis intends to be a modest basis for reflection rather than a rigid truth. Also, it raises important societal questions that go beyond this scientific framework.

Inevitably, many hypotheses, numbers, and other choices adopted in this work may strongly influence the numeric results exposed throughout this manuscript. This is due to the projective nature of the work but also the scale of the developed scenarios, which span most actors and activities involved by the electricity supply chain. However, the most notable contributions are rather to be found in the qualitative discussions based on those results, and maybe more importantly, on the relevance of the original ideas at the core of this thesis.

Executive summary

Climate change, resource depletion, pollution; the human activity of our modern and technological society has become a major source of nuisance for the rest of the planet. Humanity has a limited remaining "natural" budget to change its societal project significantly enough before consequences go out of its hands. Using that budget as efficiently as possible calls for sensible *optimization* and *planning* to ease the shift towards a more sober and sustainable project. Energy use is by far the most important contributor to these adverse observations. As it is directly correlated with economic growth, it has never ceased growing. The largest energy consumer is the electricity sector. The sector is facing boiling topics such as the ever-expanding liberalization finding in the electron new types of merchandise and the development and implementation of *distributed energy resources*, most notably distributed solar and wind-powered generation, and individual storage. To prevent the energy transition from being reduced to a pure product of economic profit that shifts pollution to areas benefiting from less spotlight, the objectives, and modalities of *electricity consumption* should be redefined.

In that context, *energy communities* have gained much attention from the economic, political, and scientific worlds recently. Such collectives gather groups of *prosumers*, i.e., proactive consumers, to act towards a common objective. Many visions are concurring for their practical implementations, from simple replicas of wholesale markets and associations of profit-seeking individuals to form large enough market players to investment settings allowing lucrative business plans. Beyond the economic unfairness of those solutions, the underlying markets governing energy sourcing and the exchanges between actors have formed a poorly legible assembly, which may prevent the desired efficiency.

This thesis aims at proposing an alternative definition of energy communities that leverage prosumer empowerment and *mutualization* of excess resources to optimize a truer reflection of electricity consumption cost. In such responsible communities, the sharing of a common infrastructure, i.e., the power grid, is also acknowledged. Non-linear network costs are thus defined, and the individual electrical loads are aggregated to evaluate the incurred costs. In addition, the members pool and

share their excess energy resources, namely the photovoltaic generation and the available storage space in their batteries, for free. In particular, several collaborative scenarios of *day-ahead energy consumption scheduling* are designed. These scenarios use the temporal flexibility consented by prosumers and the sharing of electrical resources to define the consumption schedule that will lead to optimized global costs. Several billings are defined, each promoting different behaviors and meeting different definitions of *fairness*. They are studied under the angle of *game theory*. Nash equilibrium games are formulated, and *billings* are designed, some of them using concepts of coalitional game theory.

Results are obtained based on benchmarks assuming residential prosumers, equipped with combinations of time flexible appliances such as electric vehicles and heat pumps, individual generation (solar panels), and battery storage systems. They show that such coordinated energy consumption scheduling is essential to prevent power consumption peaks and the related increased prices. Besides, some billings lead to undifferentiated prices, whereas other billings reward the time flexibility consented by individuals by either considering the actual mobilized flexibility or the pledged flexibility. Billing solutions assigning a proportion of the daily total cost of the whole community are computed using the framework of *convex optimization*. Billings distributing costs on another basis were computed by formulating suitable *variational inequalities*. All billing solutions are conveniently solved using *distributed* algorithms. Moreover, several options for coordinated investments and intraday settlement of energy exchanges are proposed to offer a complete solution of responsible energy communities.

Contents

CHAPTER 1.

Introduction, motivations and objectives

> The Earth is a fine place and worth fighting for,
>
> *Ernest Hemingway*

This year 2021 is ending with many worldwide concerns, again. Among other things, the whole world is still struggling to contain the COVID 19 pandemic, unprecedented natural disasters and disorders have struck many countries on the five continents, and the migration crises crystallize tensions more than ever. Beyond these specific facts and events, a common origin may be identified more or less directly.

Climate change and other environmental issues are increasingly leading the world into social, economic, and political turmoil. Leaders and citizens should keep in mind that the state of the planet is deteriorating dramatically and requires immediate action. The figures are clear, as reminded lately by the most recent report of the Intergovernmental Panel on Climate Change (IPCC) [1] or the World Meteorological Organization (WMO) greenhouse gas bulletin [2]. The year 2020 was the hottest ever recorded [3], greenhouse gas levels are at new records, and each year confirms, again and again, the trends about climate change. More generally, global sustainability is deeply compromised. The link between the consumption of non-renewable natural resources and other global economic subsystems, namely population, food production, industrial production, and pollution, was already described in the infamous Club of Rome's 1972 report "The limits to growth" [4]. Beyond the erroneous popular belief about a premature world collapse, the trends of the defined challenging scenarios for global sustainability indicate a significant match with the observations of the past decades [5]. It shows the obvious damaging impacts of natural resources over-consumption. The related issues now affect all humanity in its daily life, which leads to many actors calling for societal changes to limit their causes and consequences.

Energy consumption shapes our modes of living considerably. It could even be argued that it is what defines us most in modern society as it determines our scope of actions in our environment. From entirely renewable sources, i.e., animal force, water, wind, and biomass, until the late 18th century, the humans, driven by the scarcity of wood stocks and the enormous growing energy needs of the emerging industries, shifted their procurement to denser energy sources by extracting fossil fuels. Coal and later oil products have, since then, supplied most of the energy needs and unleashed unprecedented technological advances and economic growth. In the second half of the 20th century, nuclear energy mastery was the culmination of this path. Concurrently, the use of electricity greatly expanded as it is an efficient vector for the remote use of sourced energy. Electricity combined with the exploitation of ever denser energy sources led to the centralization of power generation facilities. However, fossil fuel reserves are limited and are expected to run out in the next few decades [6]. Besides, the relative skepticism about nuclear energy in some major countries suggests that possible increases of capacities would be limited. The high case scenarios of the IAEA foresees a doubling (to 20 % share of the current electricity mix) and the low case scenario indicates a status quo by 2050 [7]. These considerations imply to change production and consumption habits. Thus, the energy sector is expected to transition to a more sustainable functioning, in line with the environmental issues, and referred to as *energy transition*.

It could seem ironic that the so-called energy transition envisioned today by the main societal actors plead for the exact inverse paradigm that historically took place. More decentralization and more renewable energies; there is hence no conceptual revolution involved. Moreover, although the capital accumulated in terms of technologies allows better yields and larger-scale exploitation of these renewable resources, this is made possible at the cost of other non-renewable resources use: minerals, ores, rare earths, etc. The equation of energy generation is hence very much constrained, whichever paradigm is considered. The current efforts of the community are focused mostly on the energy sources because of their visible impact on the climate today (the energy sector accounts for 35% of the greenhouse gases emissions [8], hence making it the largest emitter), but they will more than probably be extended to the issue of the resources mass depletion tomorrow.

In this context, this thesis focuses on the use of electrical energy, which involves one-third of the total primary energy. The world's final electricity consumption is continuously rising, driven mostly by the growing demand of non-OECD[1] countries.

[1] The Organisation for Economic Co-operation and Development (OECD) is an international organization that intends to coordinate economic policies of its member countries, which are characterized by a market economy functioning.

It is expected to grow even more because of the electrification of transport, building, and industrial processes. More particularly, the energy exchanges taking place at the residential, commercial, and public services levels, accounting for roughly half of the total demand [9], are addressed.

As raised in the previous paragraphs, the most evident lever for action is reducing energy needs. However, the energy consumption level affects economic and other societal factors, which involve opinion and subjectivity. It is chosen not to address this valuable lever in such an engineering oriented thesis. This work builds on more objective considerations and aims at improving electrical operations through energy scheduling schemes. More particularly, it aims at leveraging the *engagement* of end-users to tackle the complex environmental challenges in a fair and responsible way by enabling collective-centric mechanisms such as *mutualization*. The motivations for such energy communities are further described in the next section.

1.1. Driving factors

Improving the operations of electrical power systems arises mostly from environmental concerns (pollution and availability of resources). This work intends to contribute to that effort through citizen empowerment. Engaging end-users and giving them an active role in the operations brings great potential. However, their engagement is conditioned to a fair allocation of resources and efforts. These three driving factors are hence complementary. They are further detailed as follows.

1.1.1. Environmental targets

Worldwide, governments are taking ever more stringent measures to reduce their environmental footprint. Among the pioneers, the European Union (EU) distinguished itself by adopting the "2020 climate and energy package" [10]. Its key targets aimed at cutting greenhouse gas (GHG) emissions by 20%, raising the portion of renewable energy to 20% of the total needs, and improving energy efficiency by 20% from 1990's levels by 2020. The first objective was met by the implementation of a common Emission Trading System (ETS) covering all EU's large-scale power and industry sectors, accompanied by national emission targets for the remaining sectors. However, the outperforming results should be contrasted by the exportation of the GHG emissions due to the relocation of the most polluting industry outside of Europe during that period. Altogether, the world CO_2 emissions (cf. Figure 1.1) have increased by 60% since 1990 and have not yet peaked [11].

Today, most of the world powers have set more ambitious objectives to limit global warming well below 2°C as foreseen by the Paris Agreement [13]. Through its Clean Energy Package, the EU targets a minimum 55% cut of GHG emissions by 2030 in

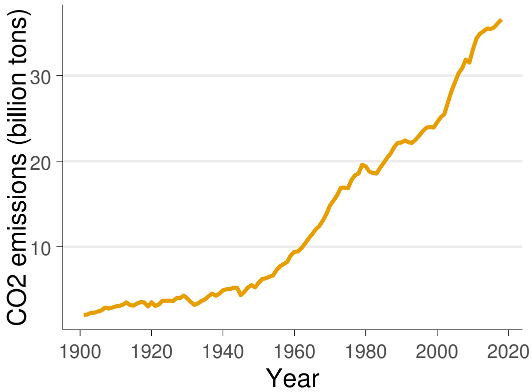


Figure 1.1.: Evolution of annual world CO_2 emissions since 1900 [12].

comparison to 1990 and envisions a climate-neutral society by 2050 [14]. China, the world's greatest emitter, pledges carbon neutrality by 2060 [15]. However, these objectives were evaluated as non-consistent with limiting warming below 2°C by the IPCC, which foresees an increase of about 3°C if no extra measures are taken. Besides, the IPCC clearly shows the dramatic consequences of a 2°C increase compared to a 1.5°C increase in a special report [16]. Therefore, even more efforts are expected to be consented to if we mean to avoid disastrous events.

Most states have certainly measured the extent of the risks incurred by global warming. However, there is much less emphasis on other environmental issues, notably the depletion of natural resources. Whereas the first issue finds a direct answer to the electricity sector by adopting a generation mix emitting less GHG, the second is often overlooked. Yet, the real cost of the transition is probably much higher than expected.

The massive deployment of renewable energies and their integration mobilize unprecedented amounts and diversity of resources. Wind turbines, electric vehicles, energy storage systems, control, and communication infrastructure all rely on the use of specific resources. On one hand, the inherent low energy density of renewable generation requires impressive amounts of concrete, steel, aluminum, and other common essential manufactured goods that use limited minerals. On the other hand, the systematic use of numerous rare earth metals in all related products (from magnetic materials in wind turbines to electronic components in smart controllers) is widely criticized because of its high environmental and geostrategic impacts [17]. For instance, Table 1.1 gives an insight of what would be the incremental commodity demand in a world with a 100 % penetration of electric vehicles (cf. [18] for details). It is often argued that the energy transition has just relocated the pollution to a few countries such as China or Latin America at the cost of a high dependency on imports.

Li	Co	REs	Graph.	Ni	Cu	Mn	Al
2898%	1928%	655%	524%	105%	22%	14%	13%

Table 1.1.: Demand increase of necessary elements, rare earths (REs), and graphite in a 100% electric vehicle world (percentage of 2017's global production).

Hence, beyond oil products, which are of utmost importance in some sectors (e.g., pharmaceutics), every mineral has potential better uses. A successful energy transition must account for the limited amount of natural resources. Therefore, arbitrage and optimization of their usage are essential. All in all, both consumption and pollution should be part of the equation to be optimized. Although it is a complex and likely imperfect task, assessing and reflecting the incurred resource consumption and their impact in terms of costs can provide a clear and straightforward variable to be optimized (cf. 2.4.3). Its popular counterpart consisting of implementing free-market mechanisms faces its inherent limitations even more importantly as exposed in 2.5.1.

1.1.2. Citizen's empowerment

The second motivation follows from the first consideration. Indeed, the environmental issues are raising awareness among the general public. The first significant effects of global warming can be experienced by most of the world today. A notable consequence is the initiation of international campaigns led by notorious environmental activists (e.g., "Youth for Climate" represented by Greta Thunberg [19]) and followed by civil society. Worldwide, many citizens wish to contribute and act for fighting the impending ecological disaster.

Furthermore, the decentralization of energy production leads to a natural reappropriation of the subject. The rapid development of personal distributed generation and the deployment of wind and solar farms also give additional visibility. Hence, there is a significant trend towards citizen empowerment, which may take a collective dimension. One successful example is the Renewable Energy Sources Cooperative (REScoop) model. It consists of a democratic business model implementing energy cooperatives where citizens jointly own and participate in renewable energy or energy efficiency projects [20]. The European federation represents over 1 million citizens grouped in 1500 cooperatives. These cooperatives are often referred to as citizen or renewable energy communities [21] (cf. details in Section 2.5), the two official definitions adopted by the EU lately in its Clean Energy Package.

In addition, the Clean Energy Package emphasizes highly on a “consumer-centered clean energy transition”. Thus, a wide range of regulations and directives, centered around improved access to energy consumption data and more accurate metering to all individuals, was adopted. Consumer empowerment is thus a leading slogan of the EU for the success of its energy transition. In line with this objective, the EU also funds related research projects. For instance, they study various ways for empowering consumers by using informative tools for different billing methods [22] or investigate efficient campaigns for increased awareness on energy savings [23]. Other initiatives include increased coordination of energy regulators (role of The Council of European Energy Regulators) for integrating consumers as full-fledged market operators.

Altogether, these initiatives encourage the general public to change the way of consuming energy, for instance, by reducing the global needs (cf. energy efficiency in Section 2.4) or by providing flexibility through their consumption level or their consumption schedule. The first acts in the long term and is mostly driven by technical advances related to the electrical appliances themselves, whereas the latter focuses on the short-term and involves the development of conceptual and technical solutions in the electricity sector (cf. demand-side management in Section 2.4). The other lever for action is the investment in Distributed Energy Resources (DERs), which also strongly influences the schemes that are involved in the consumption of electrical energy.

1.1.3. Economic fairness

All the expected changes for a more sober utilization of electricity imply much effort to devote by its end-users. Citizens’ engagement, described in the previous paragraph, is conditioned upon the economic fairness of the considered scenarios. However, fairness is a very abstract and arguable notion. On one hand, the undifferentiated treatment of actors based on their relative participation could incentivize more efforts from those having better economic positions with higher individual impact. On the other hand, a more equitable application of fairness balancing the respective economic actor weights, could mobilize a broader public and contribute to tackling energy poverty. The latter consideration is more specifically addressed through some environmental plans. Hence, the European Green Deal, the European Commission’s new strategy for achieving the CEP’s targets, defined the "Just Transition Mechanism" which gives a high dimension of solidarity and fairness to the energy transition. Ursula von der Leyen, president of the European Commission, summarized its idea by declaring: "People are at the core of the European Green Deal, our vision to make Europe climate-neutral by 2050. The transformation ahead of us is unprecedented. And it will only work if it is just - and if it works for all". More particularly, the European policy allows for the active participation of vulnerable consumers and low-income households in self-consumption and energy communities schemes.

All in all, the definition of fairness is essentially a political decision. Besides, the impact of a mechanism on global participation involves behavioral studies that are out of this scope. However, it is interesting to define different scenarios enforcing fairness in various ways. This gives more options for meeting requirements under different contexts.

1.2. Objectives

As highlighted in the previous section, defining environmental targets is not sufficient. The engagement of key actors must be leveraged and guaranteed.

This work intends to optimize the electricity sector's related resources at the low-voltage level by enhancing consumer's empowerment and a fair cost allocation. One relevant and promising mean is to gather end-users through energy collectives. Such joint platforms overshoot individual optima while allowing local realities and complementarities to be exploited, especially if they correspond to coherent electrical entities.

This thesis thus relies on such collectives to develop **responsible energy communities**. They aim to achieve a number of objectives that are developed in this manuscript.

1. **Design a collective-centric optimization structure for electricity consumption and exchanges.** To that aim, mutualization and cooperation mechanisms between members of a coherent entity are developed to mobilize maximum energy flexibility. Such an entity must correspond to the same electrical structure (e.g., one or several feeders or an entire low-voltage network) and be consistent. It can take the form of a neighborhood, an economic or industrial park, or even a whole city or region. The defined community aims to optimize the total system costs and thus acts in the interest of all its members.

2. **Recognize and account for the best image of the incurred "truth" costs.** Various varying pricing schemes are implemented in this work and used to optimize costs. Their dynamics intends to reflect the reality of the actual resources that are mobilized. They originate from the use of non-renewable energy fuels and the many resources needed for the involved assets. Although it is impossible to account for refined cost models of all externalities all at once along with the other objectives of this thesis, the proposed solutions are

intended to be generally valid and easily transposable.

3. **Develop fair distribution keys for the allocation among community members.** Prosumers' engagement can be secured only if they are billed fairly. Many different billing plans can incentivize different behaviors. Therefore, various options are proposed, compared, and discussed concerning the other objectives. Given the fact that prosumers' behavior is directly affected by these choices, there is the need for social studies and political decisions if such schemes should be put in place. The latter is, however, not included in the scope of this thesis.

4. **Give a framework that promotes responsible investments.** Considering the complexity of today's energy issues, it is very complex for small (and even big) actors to make choices that are economically sensible and environmentally responsible. The community framework helps giving better guidance to prosumers by reducing the size of the problem and brings local levers of action. The underlying engagement of communities can, in turn, be used to serve a global scheme.

5. **Put into perspective the current choices and restore common-sense.** Too many enacted choices are not discussed anymore nowadays. For instance, the relevance and efficiency of markets are rarely challenged in the engineering community. Yet they are the main drivers of today's electricity sector. Given the tremendous amounts of resources they mobilize, it is irresponsible not to stop and look at the global picture and rethink some ideas. This work intends to question some of these solutions and concepts and give another insight into the whole energy issue.

The above objectives are achieved in concrete terms by:

a Defining **non-linear pricing of grid costs** based on the aggregated load of the concerned entity and use a **dynamic pricing for the commodity costs**. Consequently, the members are interdependent through their actions. Systematically, the related (non-cooperative) games are characterized.

b Substracting the local use of DERs out of the market logic (hence excluding profits of internal transactions) by considering the **marginal use cost**. The only value attributed to electricity services is related to the use of natural resources. Concretely, photovoltaic generation and storage capacity excesses are made available to other end-users for free.

c Formulating **day-ahead energy consumption scheduling** scenarios as **convex optimization** problems.

d Developing original schemes of **excess energy mutualization and sharing**. To increase the benefits of DER investments, local members are encouraged through the billing methods to mutualize, share, and benefit from non-utilized electrical resources.

e Introducing **decentralized algorithms** for the computation of the scenarios. The main algorithms are the best response and proximal decomposition algorithms.

f Tackling the **cost distribution** among end-users issue by using the **game theory** framework. In order to ensure the compliance of the computed optimized consumption schedules, the billing methods are designed based on well-proven solution concepts such as the Nash equilibrium. In addition, several options are explored to raise the participation and full engagement of the end-users in different ways (e.g., Shapley value, VCG mechanism).

g Proposing an **investment framework** in DERs accounting for the energy exchange scenario. Several options of individual and joint investment configurations are explored. They are coupled with centralized or decentralized DER locations. The four options comply with the most advanced operational community scenario.

h Studying the options for the **settlement of the actual energy exchanges**.

Together these objectives are believed to form a coherent proposition. They will be the leading thread for the remaining chapters.

Hence, chapters 4 to 6 progressively develop scenarios that lead to the proposed responsible energy communities.

The main guidelines and hypotheses for all collaborative scenarios are detailed in 4.2.1.

1.3. Outline and contributions

In the remainder of this manuscript, the subjects are covered as follows.

- **Chapter 2** intends to provide a general background for the issues and the context that are tackled in the next chapters. Hence, the electricity supply chain is described along with its different actors and the related markets. Then, the decentralization of the electricity sector and the rising importance of low-voltage network operations are detailed. Furthermore, models of energy collectives and prosumer markets are addressed and put into perspective as pursued by objective 5.

- **Chapter 3** gives the necessary mathematical theory and tools for modeling and solving the developed schemes. Thus, some notions of convex optimization and game theory are covered. Then, some fundamentals about economic equilibria are provided. It is complemented by their characteristics, desirable properties and an insight on the main market designs.

- **Chapter 4** presents the core scenario of energy consumption scheduling based on aggregative billing. The adopted collective structure, the methodology, and the formulations are successively addressed. Then, the global scheme's performances are discussed. This chapter is largely based on the paper **"Cooperative demand-side management scenario for the low-voltage network in liberalised electricity markets"**, which was published in *IET Generation, Transmission & Distribution* [24]. The developed scenarios tackle mostly objective 2 of this work and lay the foundation of objective 1.

- **Chapter 5** evaluates, in a complemented version of the framework of Chapter 4, various cost distributions regarding efficiency, fairness, and incentivization considerations. It is abundantly based on the paper **"Pricing Electricity in Residential Communities using Game-Theoretical Billings"**, which is currently under revision in *IEEE Transactions on Smart Grid, special section on Local and Distribution Electricity Markets* [25]. It is mainly objective 3 that is addressed by this chapter, but significant complements to objective 1 are also pursued.

- **Chapter 6** develops mutualization mechanisms a step further. The increased interactions and engagement of prosumers are formalized through a community framework. Besides, a power flow formulation is introduced to better reflect the power exchange scheduling cost inside the community. These additions are also evaluated with regard to performance, fairness, incentivization, and other interesting considerations. This chapter is primarily based on the paper **"A New Cooperative Framework for a Fair and Cost-Optimal Allocation of Resources within a Low Voltage Electricity Community"**, which was published in *IEEE Transactions on Smart Grid* [26]. Objectives 1 to 3 are further completed in this chapter.

- **Chapter 7** complements the energy scheduling approach with other considerations taking place at other time horizons. Hence, the investment problem is addressed so that the energy collectives rely on efficient means. An insight into the settlement is also provided. This chapter intends to cover mainly objective 4.

- **Chapter 8** gives a final view on the whole picture. The main methodological contributions are recalled and the main findings are summarized. In addition, it delivers some perspectives before the conclusions are drawn.

Part I.

Context and Tools

CHAPTER 2.

Low-voltage networks: towards the prosumer paradigm

> Markets are lethal, if only because of ignoring externalities, the impacts of their transactions on the environment,
>
> *Noam Chomsky*

2.1. Introduction

Electricity is part of our everyday life and its importance has never ceased to grow. It is arguable that we could live the same life today without that commodity. It is indeed the backbone of our industrial society, as previously illustrated. The large-scale use of electricity began in the second half of the 19$^{\text{th}}$ century with the combined development of industrial generators such as the Gramme machine and the invention of light bulbs, which were made popular by Thomas Edison's incandescent light. The first consumers were connected to isolated systems, hence forming microgrids. However, building power plants inside the cities was not convenient because of the vast space required and the pollution caused by the generating machines. The adoption of the alternative current, made famous by the great inventor Nikola Tesla, allowed the construction of large network infrastructures and the relocation of generating units outside the cities. This also led to the interconnection of the networks and made electricity a key element of the globalization process in the 20$^{\text{th}}$ century.

The turn of this millennium marked a disruption for the electricity sector. The liberalization of the electricity markets was a major milestone initiated by the

EU to ensure the functioning of the internal energy market [27] and integrate state members a step further. Initially, production, transmission, distribution, and electricity trade were ensured by a single entity, which was usually a state or a regulated private monopoly. It was decided to open the generation and trade to a competitive market and separate them from the network operation, which remains a regulated monopoly.

Another disruptive trend that has been taking place in the last two decades is the massive deployment of Renewable Energy Sources (RES), led mostly by wind and photovoltaic technologies, and the more recent development of affordable Energy Storage Systems (ESS). Together, these Distributed Energy Resources (DERs) are expected to be a major driver of the energy transition. However, their implementation is reshaping the operation of the whole grid. The fluctuating behavior of renewables and their local integration at low-voltage and medium-voltage levels lead to global decentralization and the need for increased coordination.

Together, these economic and technological changes have triggered the development of many active management schemes and a variety of new business models on the demand-side. In particular, Demand-Side Management (DSM) programs aim at improving electricity consumption and align it with the modern generation reality. Their combination with consumer-centric approaches by better capturing the opportunities arising at that level through individual empowerment is expected to be a real game-changer and lead towards a prosumer paradigm.

The remainder of this chapter further develops the current electricity supply chain and its associated markets. Then, the decentralization and the new consumption schemes impacting the demand-side are introduced. Finally, an insight on energy collectives and their different competing business models is provided along with the outline of this work's proposal.

Note that it is chosen to adopt a European perspective by default, without loss of generality. Indeed, most liberalized electricity sectors share many similarities, so that this work is easily extendable to other regions.

2.2. Electricity sector map

In a traditional electricity system, most power is generated by large power plants connected to the transmission grid (managed by the transmission system operator). The high-voltage system carries the energy over long distances and then feeds a geographical zone through the distribution grid (operated by the distribution system operator) that converts electricity several times at ever lower voltages until reaching the end-consumer.

From a commercial point of view, electricity producers can either sell their energy to suppliers on different electricity markets or directly act as suppliers that, in turn, retail the electricity to the end-users. The electricity balance is ensured physically by the transmission system operator and commercially by balancing responsible parties. These main actors' roles are further described in Figure 2.1 and hereafter. In addition, a particularization to the Belgian landscape is provided in Box 2.1.

Note that only producers, suppliers, and consumers operate in a liberalized regulatory framework, whereas system operators remain regulated monopolies.

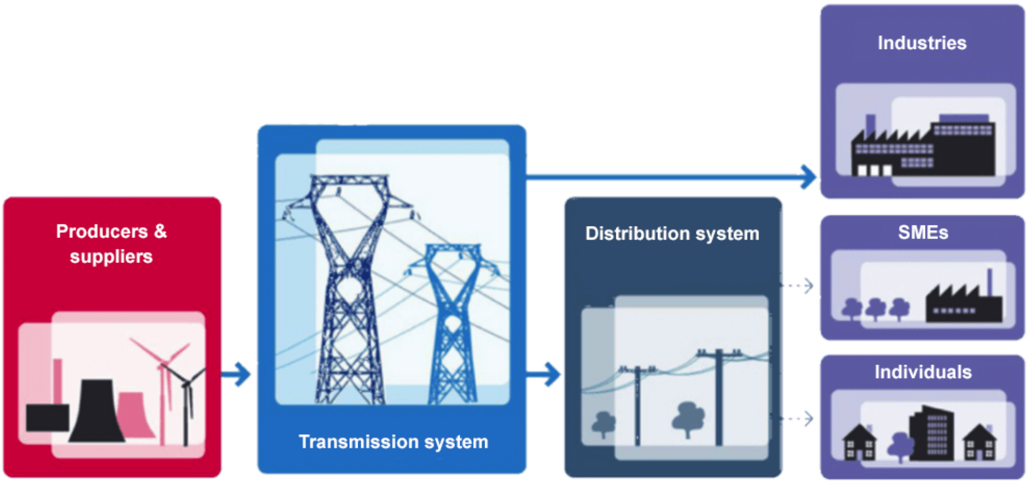


Figure 2.1.: Map of the physical electricity supply chain and its main actors. Source: [9].

2.2.1. Producers

Producers are private companies that generate electricity by operating large power plants (e.g., nuclear, gas, coal, or biomass-fired units) or renewable farms powered by the wind or the sun. Although end-users do not necessarily buy it directly from them, producers can also act as electricity suppliers, whose role is further detailed. Otherwise, they can sell their production to the wholesale markets through a balancing responsible party. Besides, production units can also provide some generation flexibility that is mobilized by the system operators (cf. ancillary services in 2.3.4).

2.2.2. Transmission system operators

The transmission grid includes the high and very high-voltage lines, typically 110 kV and above[1], that transport electricity on a national and international level. It is managed by the Transmission System Operators (TSOs) who have the four following core missions:

i) They own, maintain and invest in the transmission infrastructure in order to integrate the development of new generation facilities and interconnect the neighboring countries.

ii) They operate the transmission system and maintain the generation and consumption balance by monitoring and coordinating the actors.

iii) They guarantee the quality and the security of supply.

iv) They act as market facilitators and ensure that all players have equal access to all resources necessary for their trade.

Given their natural monopoly (only one grid infrastructure for a defined area), they are strongly regulated by governments.

2.2.3. Distribution system operators

The Distribution System Operators (DSOs) manage the electricity distribution infrastructure, which encompasses all the regional and local networks (i.e., medium and low-voltage lines) supplying the end-users. Each DSO has, therefore, a regulated monopoly on a defined area. Their main responsibilities are:

i) They own, maintain and invest in the distribution grid infrastructure, which connects most final end-users.

ii) They operate the network and convert electricity from higher voltages (transmission grids) to lower voltages and distribute it to consumers.

iii) They perform most of the metering activity (households and most businesses).

iv) Possibly, they can fulfill public service missions such as managing the road lighting or providing energy to protected consumers.

Note that it is common for DSOs to act in the same role for the gas distribution network infrastructure.

[1]Some TSOs manage lower voltage lines (e.g., Elia, the Belgian TSO, manages power grids of 30 kV and above.)

2.2.4. Suppliers

Electricity suppliers are companies that either own generation means and/or procure energy on electricity markets (through a balancing responsible party) to sell contracts to end-users. This is the main activity that was impacted by the liberalization of the electricity markets. Today, individuals or businesses can freely choose their supplier and one of the pricing plans they offer. Each contract has a certain length and can have specific conditions such as a label of origin (e.g., 100 % renewable sources). Suppliers are also responsible for the billing, which compensates the other implied parties (TSO, DSO, public authorities, etc.).

2.2.5. End-users

Historically, end-users have been mostly passive consumers ranging from residential homes to big industries. They procure their electricity needs through the main grid by contracting a supplier. With the widespread development of distributed generation (e.g., photovoltaic panels, wind turbines), some of them now produce part or all of their electricity locally. Besides, the emerging demand-side management and consumer-centric schemes (cf. sections 2.4 and 2.5) tend to empower consumers and make them respond to some signal through their consumption and production course of action. These proactive consumers are termed as *prosumers*[2].

2.2.6. Balancing responsible parties

Another major player in the liberalized electricity supply chain is the Balancing Responsible Party (BRP), which can be either a producer, an important industrial company, a supplier, or a trader. BRPs are each responsible for a portfolio of one or several access points to the transmission grid for which they must ensure the equilibrium between the offtakes, the injections, and the commercial exchanges with other BRPs. Indeed, the balance at the system level between electricity production and consumption must be guaranteed at all times. They resort to various electricity markets to cover day-ahead and intraday operations, and they are financially responsible for the potential imbalance. Usually, it is the suppliers that have the role of BRPs for small end-users whereas large consumers and producers can ensure these responsibilities.

2.2.7. Aggregators

Aggregators are third parties that combine a set of small decentralized production and consumption units through virtual power plants. Such portfolios of coordinated

[2] Another more general definition considers prosumers as end-users that own production means.

Box 2.1: A focus on Belgium

Elia is Belgium's only TSO. It is responsible for managing the whole Belgian transmission grid, encompassing 380, 220, 110, 70, 36 and 30 kV lines. In parallel, several regional DSOs, namely Fluvius (Flanders), ORES (Wallonia), Sibelga (Brussels), RESA (Wallonia), and others, operate distribution networks, which include a wide range of medium-voltage and low-voltage (230/400 V) lines. Belgium is sourcing most of its electricity from nuclear generators (around 47% of the total electricity) although they are planned to be closed down by 2025 and replaced by gas turbines. The rest of the electricity is sourced mostly from gas (± 28%), coal (± 3%), wind (± 10%), sun (± 4%), biomass (± 4%), wastes (± 3%), and hydraulic (± 1%).

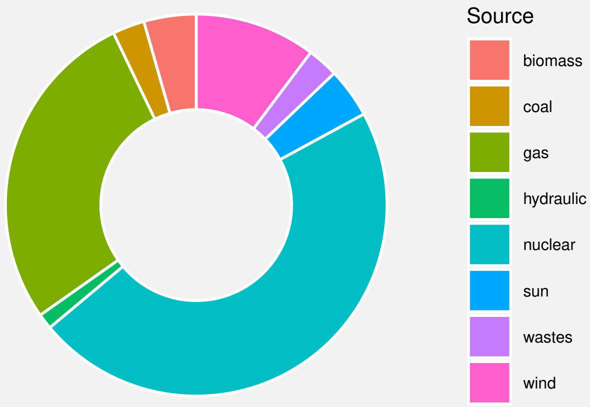


Figure 2.2.: Belgian generation mix. (source: Elia 2019)

On a regular basis, Belgium is depending on imports and a lot of investments are therefore made on expanding cross-border capacities. In particular, Elia and its partnering TSOs have achieved the construction of HVDC[a] links connecting the UK (NEMO) and Germany (ALEGRO) and is planning a new interconnector with Denmark. Belgium is integrated in the EPEX SPOT market encompassing most western European countries (Germany, Luxembourg, France, the Netherlands, the UK, Denmark, Finland, Sweden, Norway, Austria and Switzerland) and a single bidding zone covers the whole country. The federal regulator, the CREG, is responsible for supervising and monitor the application of policies while the regional regulators (VREG, CWaPE and Brugel) are responsible for enforcing the operating rules of liberalized markets functioning.

[a] High-Voltage Direct Current (HVDC) lines are used to interconnect two electrical nodes that are too distant from each other for a regular AC line. Electricity conversion units are necessary at both ends of the line.

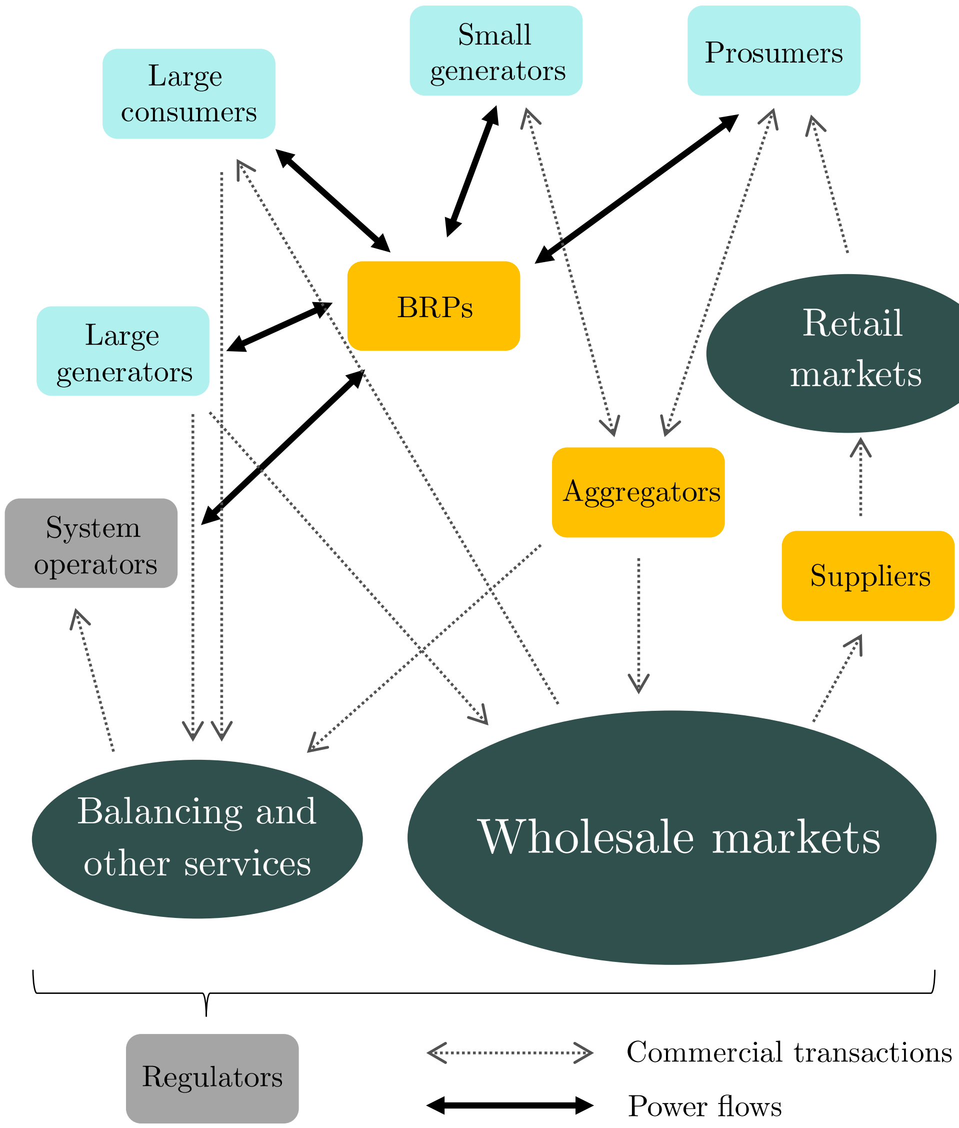


Figure 2.3.: The main actors and their interactions in the electricity markets. Light blue boxes correspond to liberalized actors holding physical assets, yellow boxes are roles (several roles can be) and Commercial transactions may be ensured by third parties surch as traders. Note that BRPs only coordinate the power flows.

units thus have a significant enough size to propose balancing services and trade energy on electricity markets.

2.2.8. Regulators

Regulators ensure that transparency and competition rules are satisfied by market players. They aim at preserving the general interest and align the practices with the regional, national, and international policies. They often have the additional role of counseling public authorities on energy-related issues.

2.2.9. Summary

Figure 2.3 summarizes the main interactions existing between the actors and the main markets (cf. detail in the next section). Balancing responsible parties coordinate their portfolio to ensure the equilibrium between production and consumption (power flows). In the event of a mismatch, the TSO activates reserves, which were contracted on dedicated markets. These balancing as well as other services are offered by large generators or consumers, or intermediaries (e.g., traders). Wholesale markets are fed with offers originating from large generators, aggregators, and intermediaries. Suppliers procure energy on these markets and offer consumption plans to small end-users. Finally, regulators supervise the whole sector and its actors. The actors in yellow (BRPs, aggregators and suppliers), are not physical. Their role can be endorsed by any unregulated actor (e.g. large consumers and producers) or a third party.

2.3. Traditional electricity markets

Deregulation has turned electricity into a good that can be traded almost like any other commodity (cf. Section 2.5). However, given the physical nature of electricity, there is a wide variety of different markets to answer different needs at different time horizons, and ensure a proper operation of the electricity supply chain.

2.3.1. Wholesale electricity markets

These are the markets where producers' offer meets suppliers' demand through their respective BRPs. Two main types of markets can be differentiated: *over the counter* (OTC) markets and *power exchange* (PX) markets [28]. Over the counter markets consist of private bilateral trades relating to many different scales, time horizons, and specifics. They are less regulated and tend to be less transparent. Power exchange markets involve multilateral and anonymous transactions of standard energy block bids through an auction system. They are transparent and operated by a centralized market operator. Depending on the time horizon, trading energy volumes can be addressed differently.

i) **Long-term markets.** Most of the energy volumes are secured well in advance (up to a few years) and are usually contracted on dedicated OTC markets (forward contracts), although they can also be obtained on PX markets (a.k.a. futures markets). The suppliers need to safeguard a significant portion of their portfolio in order not to be exposed to high fluctuations in prices. Over time, this portion tends to decrease in favor of shorter-term markets, under the pressure of the energy-intensive industrial sector that hopes to benefit from better prices [29], and due to the fluctuating nature of renewable generation units.

ii) **Day-ahead markets.** Day-ahead market trades usually represent a significant volume of the total energy and allow to adjust the long-term position of BRPs. For each hour of the following day, prices are cleared on PXs based on a merit order mechanism. All the offers and bids are aggregated and ranked through two curves. The intersection corresponds to the clearing price, i.e., the marginal price of the marginal offer. All accepted bids are paid that marginal price (cf. Box 2.2). Note that next-day delivery contracts can also be negotiated bilaterally (through OTC markets).

iii) **Intraday markets.** BRPs can rectify the balance of their portfolio on the same day of delivery by trading electricity on intraday markets. These usually consist of organized OTC transactions that are continually cleared on PXs. Note that trades can take place up to a few minutes before delivery, but it is, however, impossible to exclude imbalance in due time. The latter is eliminated by balancing mechanisms, which are managed by TSOs.

Remark 2.1. In some cases, adequacy cannot be guaranteed because of the local lack of a clear signal for investment. In such cases, remuneration mechanisms may be implemented for generators to invest in sufficient capacities (cf. Belgian example in Box 2.4).

2.3.2. Retail electricity markets

Retail markets include a number of competing suppliers that want to sell electricity contracts to end-users. Their offers are usually under the form of a fixed or variable commodity rate that is defined for a certain period of time. It is worth mentioning that as much as more than half of the final bill is not subject to competition because it results from grid fees (received by TSO and DSOs) and various taxes which are regulated or imposed by the states. The example of a typical Belgian electricity bill is provided in Box 2.3.

2.3.3. Cross-zonal capacity markets

Electricity markets can't ignore the physics of power flows. In the same bidding zone, i.e., a geographical entity in which no transmission constraints are considered

Box 2.2: Electricity spot pricing

Beyond the liberalization process and the related political decisions and events, the formalization of electricity as a commodity intervened following the work from Fred C. Schweppe, Michael C. Caramanis, Richard D. Tabors, and Roger E. Bohn. They defined in [30] the concept of *Locational Marginal Price* (LMP) to enhance spot pricing of electricity. Most current electricity markets (day-ahead, intraday, reserve, etc.) were established based on that theory, which for each location on a transmission grid, imputes a single price corresponding to the marginal cost of supplying an additional unit of demand at that location. It is, in practice, obtained through a global optimization problem that accounts for the operational constraints (transmission, generation, loads, etc.). Most US marketplaces are directly based on LMPs, a.k.a. nodal pricing. In the EU, zonal pricing is enforced. It adopts a very similar framework but computes uniform prices for a whole electrical zone (i.e., broader than a node). Besides, operational constraints are only partially included through a simplified flow-based model [31]. However, they can be directly integrated into complex bidding blocks in the optimization problem. Spot pricing is considered to be an appealing market mechanism because it features two desirable economic properties: individual rationality and budget balance. The first ensures that an agent can consistently achieve as much expected utility from participation as without participation, and the second holds if the equilibrium net exchanges are non-negative. They are further detailed in Section 3.4.

for the local trades, possible congestions on power lines, and their corresponding costs, are handled by the TSO (they are socialized through grid fees). On the other hand, when trades involve two different bidding zones, they must satisfy the cross-zonal capacity allocation to avoid congestion. The cross-zonal capacity allocation of market players is defined either implicitly by the day-ahead and intraday markets (energy and transmission capacity markets are coupled) or explicitly by trading transmission rights on forward cross-zonal transmission capacity markets.

2.3.4. Balancing markets and other ancillary services

Due to the many constraints of operating electrical grids, many coordination mechanisms must be put in place. They are often referred to as ancillary or system services.

The most straightforward constraint is the balance between production and consumption of electricity. The first must be sufficient for the second but not exceed it because it is not possible to store electricity on a large scale. A mismatch between production and consumption leads to frequency changes (the equilibrium point in

Box 2.3: Typical Belgian electricity bill

An estimation of the electricity bill components provided by ORES [32] (walloon DSO) gives the following numbers based on a residential electricity bill of 1000 €:

- *Supplier*: 320 € (the amount of this item varies considerably along time depending on the current electricity markets conditions.)
- *TSO*: 44 €
- *DSO*: 227 €, of which 186 € for managing the network, 16 € of income taxes and 25 € for shareholders.
- *State*: 409 €, of which 46 € for energy policies enforcement, 171 € of subsidies to renewable energies and 192 € of taxes.

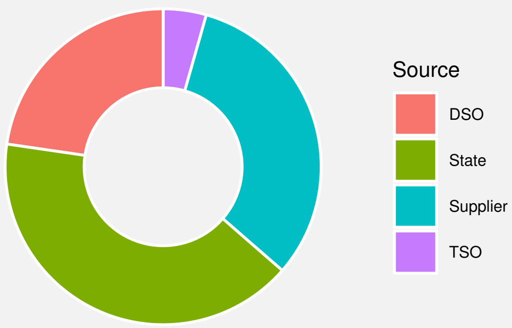


Figure 2.4.: Items of the electricity bill.

All components of the bill for LV end-users are energy-based, i.e., they are proportional to the consumed energy (recently, a capacity-based prosumer tariff on injection power was defined). In contrast, end-users connected to MV or HV substations are subject to capacity-based tariffs for the grid components. Prices are defined based on their power profiles over a certain time period. Usually, it is the maximum or mean capacity used over a monthly or annual period.

Europe is at 50 Hz). If too much electricity is produced, the frequency increases, which may lead to interruptions of generation units and loads. If not enough electricity is fed into the grid, the frequency drops and may lead to a complete collapse of the system, i.e., a blackout.

Although BRPs make all the possible arrangements on the previously described markets to have the best balance possible, they can't predict the exact consumption or face last-minute unforeseen events (production loss). The real-time imbalance is corrected by the TSO through the activation of operating reserves. Those are energy resources provided by balancing service providers that are negotiated on specific markets:

i) **Reserve capacity allocation markets**. The negotiated contracts remunerate the available capacity kept at all times regardless of whether it is activated or not.

ii) **Balancing electricity markets**. The contracts remunerate the actual activation of the reserve capacity and track the mobilized energy.

Box 2.4: The Belgian capacity remuneration mechanism

Like several other EU member states, Belgium has been heavily subsidizing investments in renewable energy sources. Conventional, fossil fuel-based power plants, which ensure the adequacy of the system by balancing renewables' stochastic generation tend to be unprofitable. It is aggravated by the phase-out of nuclear energy, accounting for the largest share of the Belgian generation mix (cf. Box 2.1), that is planned for 2025. The *Capacity Remuneration Mechanism* (CRM) is an instrument aiming to complement energy markets with a capacity market that guarantees the availability of sufficient capacity to ensure electricity supply [33]. It is hence the base for the future adequacy of the Belgian energy system. The CRM market involves auctions for the upcoming 15 years. This mechanism replaces the strategic reserve, which was not providing sufficient incentive for new capacity investment. In Europe, other countries such as Italy, Poland, the UK, Ireland, and France have similar mechanisms, but the Belgian CRM is one of the most supportive in Europe for new build.

Some balancing markets include both remunerations. Moreover, different products addressing different times of reaction usually coexist. The most important balancing products are the following.

i) **Frequency Containment Reserve (FCR).** Its goal is to limit frequency deviations and avoid a blackout. It should be active within 30 seconds in each participating unit.

ii) **Automatic Frequency Restoration Reserve (aFRR).** It is activated by the TSO within 5 minutes. It is thus coordinated, and regular setpoints are sent to the participating units.

iii) **Manual Frequency Restoration Reserve (mFRR).** This reserve aims at facing a large and prolonged imbalance. The energy at stake can thus be consequent.

Note that in Belgium, these products are known as primary control (R1), secondary control (R2) and tertiary control (R3) respectively.

Many other ancillary services address other grid constraints. Among them, the main ones are the following:

- *Reactive power and voltage control.* In order to maintain the voltage within an acceptable range, the absorption or production of reactive power flow is remunerated through dedicated contracts with the TSO.
- *Re-dispatch reservation.* Market outcomes can occasionally result in physical violations on the grid (congestion, voltage limit, etc.) because the market clearing process does not include the operational constraints (unlike the nodal pricing applied in the USA). Currently, generators are legally obliged to participate in the re-dispatch and compensated at a regulated price but the Clean Energy Package intends to organize a dedicated market in the future.
- *Restoration service.* TSOs usually conclude contracts for the provision of a black start capability. In the event of a blackout, the contractors must be able to energize the grid and help the TSO restore the electricity supply.

2.3.5. Imbalance settlement

As previously mentioned, although the many different markets aim together at balancing production and consumption, the system always faces some remaining imbalance which must be covered by activating the reserve (cf. 2.3.4). The imbalance settlement consists of quantifying each BRP's actual imbalance and then, pass on the costs related to the reserve activated by the TSO. If the imbalance of BRPs is increasing or decreasing the total system imbalance, they pay or receive a financial contribution. Imbalance settlement is, therefore, a key element for a BRP's market strategy because they tend to play with their imbalance to improve their position [34]. One will note that it can be a source of inefficiency.

2.3.6. Summary

As developed throughout this section, the market layout represents a complicated succession of transactions of different natures and time horizons (cf. Figure 2.5). It is even more complicated when the particularities of each country or bidding zone are considered. However, there is a strong trend towards more integration and harmonization in the EU. There are three main market products: energy,

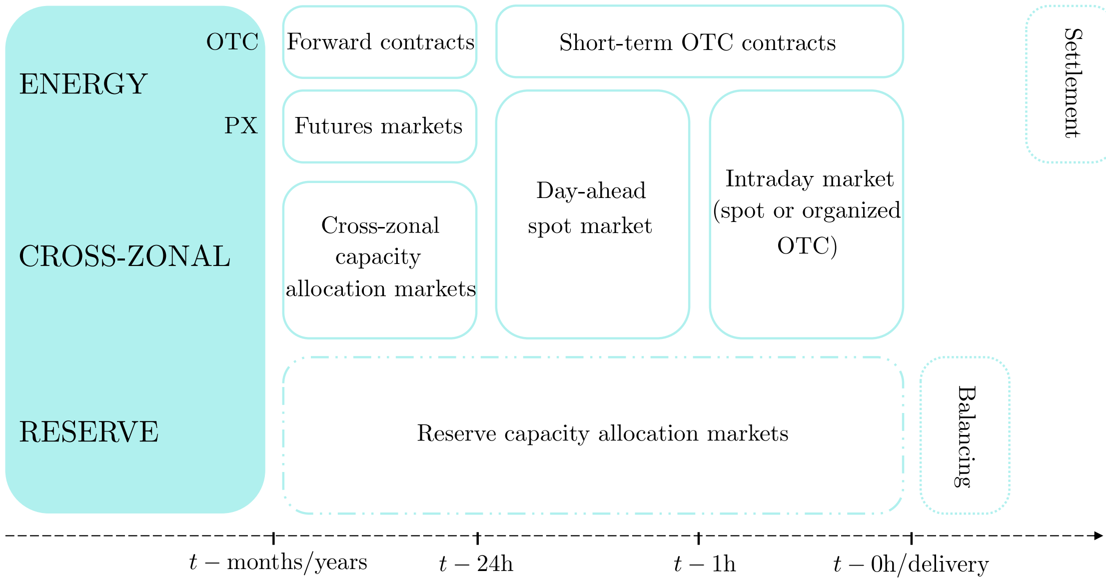


Figure 2.5.: The main electricity markets and their time frames.

(cross-zonal) transmission capacity, and reserve. Energy and transmission markets are organized separately on a long-term basis but are coupled for the day-ahead and intraday, i.e., energy markets are cleared while taking into account cross-zonal capacities. Reserve markets encompass many different functionings and time windows depending on the country, although the latter in the EU tend to reach the day-ahead horizon. Instantaneous energy imbalance is corrected by the TSO through activating the previously contracted reserve (balancing). Then, the energy imbalance of each BRP is assessed and financially settled by the TSO.

2.4. From centralized to decentralized system operations

2.4.1. Triggers of change

Historically, electricity networks and the supply chain were designed to connect large production plants to the end-users. Power would flow from higher voltage to lower voltage levels. To provide improved security of supply, higher voltage lines, which transport bulk power flows on long distances, are arranged in a meshed pattern. On the other hand, distribution lines and cables usually form radial layouts because an interruption at that level affects much fewer consumers and is often not critical. They are referred respectively to as the *grid* and *network* topologies.

However, concurrently with transactions and actors-related changes, there have been significant technological advances that are redefining physical operations. The main factors of change that can be highlighted are the following.

i) **Electrification of the loads.** Beyond the energy demand of emergent countries, which is expected to rise, there is a trend towards the shifting from more intensive carbon-emitting sources to electricity by energy-consuming applications. Hence, a substantial portion of the transportation fleet is adopting electricity as its primary energy vector. The most notorious application is the *electric vehicle* (EV), popularized and first produced on a large-scale by the Tesla company and followed, since then, by all the major manufacturers. In the last few years, the EV market has taken a colossal leap forward. It accounts for over 6.3% of the overall car sales in the first half of 2021 [35], and there are significant commitments of governments and companies to reach ambitious targets. Hence some leading manufacturers consider the electric option for all their models in the medium term (e.g., achieved by Volvo in 2019 and foreseen in 2030 for Volkswagen). Besides, the EV30/30 IEA plan, already adopted by countries such as France, Germany, the Netherlands, China, Canada, or the UK, foresees a share of EVs summing up to 30% by 2030 [36]. Space heating is another major contributor to the increasing electrification through the massive deployment of domestic *heat pumps* (HPs). Currently, HPs provide heat in about 10% of all buildings in Europe, but the market achieves growth of more than 10% each year and represents the majority of heating devices installed in newly constructed buildings in many countries [37].

ii) **Decentralized generation.** The beginning of this 21$^{\text{st}}$ century has seen the large-scale deployment of renewable energy sources powered by the wind and the sun. The climate targets set by governments and organizations (e.g., Clean Energy Package of the EU) ensure continuous penetration growth. Given the low energy density of these sources, such facilities usually have much smaller power capacities and have the advantage of being able to be installed closer to the load they serve through the distribution grid. Wind turbines are mostly connected to the medium-voltage (MV) network by generation companies and large businesses, and solar (photovoltaic) panels are usually installed on the low-voltage (LV) network by households, public buildings, and small and medium businesses. However, large solar farms connected at MV level are developing as well. This ramping shift of the generation mix introduces however many new challenges and issues. Among them, the intrinsic uncertain nature of wind and sun raises the issue of availability. Whereas conventional power plants are usually partially or fully controllable, the wind speed and the solar irradiance fluctuate along with the meteorological events.

iii) **Storage and multicarrier energy systems.** An increasing trend consists of storing electricity by electrochemical means or by coupling electricity with other energy vectors to create a buffer effect. To this day, battery storage systems have undergone significant improvements and costs decreases, and they are starting to be deployed on a large-scale. However, another promising approach is converting electricity into a high energy density gas such as hydrogen or methane. This process is known as power-to-gas (P2G) technology. Generally, it is much easier to store energy as gas because of the line pack effect in existing pipelines, or the easy use of large storage tanks. This possibility and the easy reverse conversion allow increasing the overall flexibility. They could also have a significant impact on the way that the respective markets are operated and lead to their coupling.

iv) **Intelligence, computation, and communications.** Advances in many information fields such as *artificial intelligence* (AI), robotics, the Internet of Things (IoT), cloud computing, smart sensors, and other technologies have initiated a new industrial revolution with tremendous consequences on most business sectors. These emerging technologies are expected to substantially benefit the electricity sector, which deals with ever-growing information sources and levers of action. Connectivity is thus expected to enable the *smart grid* concept. Under the described changing factors (new loads, decentralized generation, expanded flexibility), smart grids aim at facilitating the exchange of information between the different actors and optimize electrical operations. They are starting to make their way through the mass deployment of smart meters and smart appliances. Indeed, the former allow communication and interaction with end-users, and the latter usually feature automation potential. Smart grids are closely related to broader concepts such as smart cities that aim at collecting data of many types of sources to manage assets, resources, and services as efficiently as possible.

All these trends concur in changing the paradigm in place. From a centralized and top-down operation of the electricity supply chain, there is a shift towards more decentralized schemes where end-users and the distribution network occupy a more prominent place. A significant benefit of this adoption is the opportunity to leverage flexibility on the demand-side. All those programs that aim at modifying the consumer energy demand are referred to as Demand-Side Management (DSM).

2.4.2. Demand-side management and Load management

The relative decrease of controllability in the electric system has a direct impact on *adequacy* and *security of supply*. The first refers to the the electricity generation system's ability to comply with demand requirements at all times, and the second refers the ability of an electricity system to withstand sudden disturbances such as electric short circuits, unanticipated losses of system components or load conditions

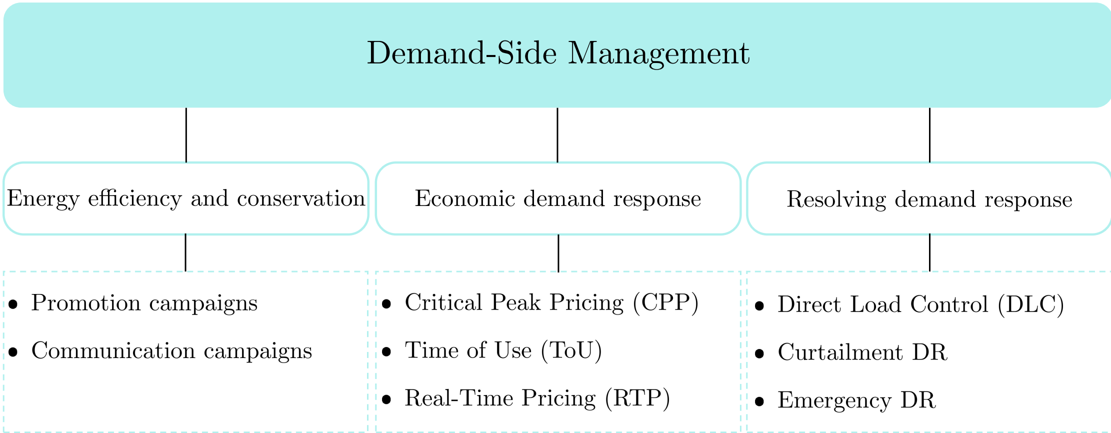


Figure 2.6.: Classification of demand-side management programs.

[38]. Control affects reliability (encompassing both adequacy and security of supply) because it must ensure the equality between production and consumption at all times.

The lack of controllable generation can be addressed by proactive and coordinated means at different levels. Either it can be managed on the supply-side by implementing backup solutions (e.g., peaking generation) or large-scale storage, or it can be tackled by leveraging flexibility on the demand- side. The latter solution has tremendous potential and receives considerable attention from all stakeholders because it is more cost and environmentally effective. Demand-Side Management (DSM) originates from a US government policy (National Energy Conservation Policy Act, 1978) that aimed at reducing the need for utilities[3] of supply-based power planning because of its high capital investments [39]. Today, DSM programs refer to a wide range of measures that modify the consumer load profiles to answer the joint reliability requirements of power systems [40]–[44]. The leitmotiv of DSM is the coordination of the demand-side through campaigns, optimization, or controlling schemes. DSM programs can be classified according to many criteria. We propose categorizing them into different families based on the time horizon and the related purpose (this is not an exhaustive overview and other classifications are possible), cf. 2.6:

i) **Energy Efficiency and Energy Conservation.** Energy efficiency aims at decreasing the overall load by improving the technologies of end-use appliances [45]. The programs usually consist of promoting the use or the replacement

[3] A utility is an integrated entity that usually owns and operate the whole electricity supply, from generation to supply, in a defined area. They are still the most common organization in the USA.

of specific devices (e.g., switch light bulbs to LED lighting) that consume less energy for the same service. Other awareness and communication campaigns directed towards energy savings by adopting adequate behaviors (e.g., turn off the lights when unnecessary) are referred to as energy conservation programs.

ii) **Economic Demand Response.** It consists of measures that aim at shaping the load using financial incentives. The main motivations are the adaptation to the generation conditions (adequacy) and the minimization of costs. The leading signal provided to the end-users is, therefore, price. The needs of the involved system actors (mainly suppliers and system operators) are reflected through dynamic pricing schemes. There are two main ways to execute a dynamic pricing strategy; either time-based or capacity-based. The former is relevant because the power system conditions vary a lot through time. The preeminent time-based pricing schemes are the following.

- *Critical Peak Pricing (CPP)*. When critical events of peak energy demand occur, such as during a cold winter day, an increased rate is applied to encourage end-users to delay or reduce their electricity consumption. These events are usually limited to a few activations per year, and consumers are warned (e.g., one day ahead).
- *Time of Use (ToU)* consists of offering a plan with various prices for specific time blocks. This allows to better adapt to the general periodicity of the overall offer and demand by letting end-users adjust their consumption (possibly through automation) to reduce their expenses. ToU schemes unlock flexibility for more efficient electrical operations. However, they remain in force for extended periods (e.g., one season) and, thus, cannot rectify daily events. One notorious and widespread example of basic ToU is the day/night tariff, which differentiates prices between "day" hours and "night" hours.
- Finally *Real-Time Pricing (RTP)* adopts variable rates for variables time blocks (up to real-time) in a single day. It is what can reflect wholesale market conditions as closely as possible. Such pricing schemes are evidently most relevant if end-users hold scheduling flexibility and have control means.

These pricing schemes are enforced either by utilities or by the different actors of the liberalized electricity sector. In the latter case, a supplier can better reflect the market conditions by giving a signal to its customers through the applied rate, and a system operator could incentivize the avoidance of peak loads that are known to occur at certain hours. However, a more effective way to avoid peak loads consists of adopting capacity-based pricing. *Capacity-based pricing* encompasses tariffs that charge individuals as a function of their power level consumption. Although most current capacity-based tariffs are usually limited to the maximum or mean capacity used over a monthly or annual period, much more granular schemes can account for times blocks of one hour, a few minutes, and up to a few seconds. Such pricing is not commonplace yet, but it has clear potential for grid operators to better reflect their incurred costs.

iii) **Resolving Demand Response.** These are directing actions on the load that are triggered in the short-term to ensure proper security of supply. They are also referred to as incentive-based DR because they are usually financially compensated through a contract or a successful bid on a specific market. Some of the main techniques are:

   - *Direct Load Control (DLC).* Upstream actors such as a utility, a supplier, or a system operator can directly access and remotely control some end-users appliances. These are mainly residential consumers or small and medium businesses. Industrial consumers have specific processes that usually aren't suitable for this technique.
   - *Curtailment DR programs.* End-users can sell their energy consumption flexibility by shedding some of their load when required and under certain conditions.
   - *Emergency DR programs.* Many different contracts exist for end-users to help restore a normal service of grid operations.

   Note that these contracts do not replace electricity supply contracts. In the EU, these programs are more organized and consistently integrated than in many countries thanks to wholesale, transmission, balancing, and ancillary electricity markets (cf. 2.3).

Generally speaking, the EU is very active in DSM-related programs. Various regulations enforcing both energy efficiency and demand response programs are detailed in article 15 of the Energy Efficiency Directive [46]. Today, demand-side management, together with other investment and operational optimization programs, finds the broader definition of *load management.* This definition also has the advantage of being less US-specific. Load management, thus, aims at optimizing the investments and operations of a defined entity by enabling controllable flexibility. The flexibility can be of different kinds:

i) **Energy level.** Consumers and prosumers can structurally consume and/or produce more or less.

ii) **Energy schedule.** Consumers can consent to more or less scheduling flexibility for some of their loads.

iii) **Energy storage.** Consumers can charge or discharge energy through a personal battery.

As previously mentioned, this thesis does not address the energy level because it is mostly related to behavioral functions and political will. It is, however, the lever that probably holds the best potential to answer the needs of an energy transition with a significant enough impact (cf. [16]). The other flexibility sources, which are tackled by the solutions proposed later, mean to get the most out of a sustainable energy consumption level.

2.4.3. Energy exchange scheduling

Previously, a number of levers for enhanced control on the demand-side (demand response programs) were introduced. Beyond their crucial needs for system reliability, they can be employed to increase the coordination among actors, and aim at operating optimally all related resources (e.g., optimize local power flows and DERs usage).

A well-designed economic optimization can align with such an objective. It has the advantage of being very convenient because it uses cost as a common denominator. If the right price signal is provided, it is possible to take the best actions for a specific time horizon. Dynamic pricing programs, when they are combined with energy flexibility, emerge as an obvious choice. Hence, **energy exchange scheduling coordinates end-users to optimize the cost of their energy exchanges (consumption and sharing flows) using their temporal flexibility in a defined entity**. The two most relevant problems are long-term investment planning and day-ahead scheduling. This work focuses mostly on the latter, though it gives an insight on the first in Chapter 7.

The one-day ahead horizon for energy exchange scheduling is highly relevant and convenient. On the one hand, individuals usually have a clear view of their needs for the following day, and on the other hand, it coincides with the day-ahead electricity markets where most of the balancing efforts are contracted. Scheduling one day in advance is also a good trade-off because a shorter optimization window would not give much flexibility, whereas a longer one would deteriorate predictions and increase the risk of scheduling changes.

There exist many energy exchange scheduling programs. The main differentiating characteristics are:

i) **Action perimeter.** An energy exchange scheduling scheme applies to a defined physical entity. There is no restriction on its extent, but it is usually in line with a coherent economic or physical entity. Some of them are the individual, a microgrid, a community, a low-voltage feeder, or a business park.

ii) **Decision scope.** The scheduling includes a set of flexible appliances. They can include devices for which time flexibility can be granted (e.g., electric vehicle, heat pump, washing machine, etc.) and energy storage systems (e.g., personal battery).

iii) **Cost accounting.** The price signal should reflect what is meant to be optimized. If, for instance, power losses should be accounted for, the overall price function should increase non linearly because these losses depend on the square of the line currents.

iv) **Cost Distribution.** Usually, the enforcement of the energy scheduling solution is conditioned to the cost distribution among participants. Defining an efficient and fair distribution key is essential to mobilize maximum engagement.

v) **Computation.** There are usually various ways to implement the decision algorithm. There is, however, a growing trend for adopting distributed algorithms. It is better suited to the increasing decentralization and the emergence of computation capabilities directly on end-users, as well as handling data privacy concerns.

Remark 2.2. In chapters 4 and 5, the optimizing processes are referred to as *energy consumption scheduling*. It can be viewed as the particularization of energy exchange scheduling to entities that do not exchange energy (willingly) between members. The exchanges are restricted to providing coordination grounds (information, computation, etc.). Energy exchange scheduling thus has a more engaging scope that is developed by this thesis in the form of energy communities (cf. 2.4.3 and Chapter 6). Figure 2.7 depicts the difference between the two schemes.

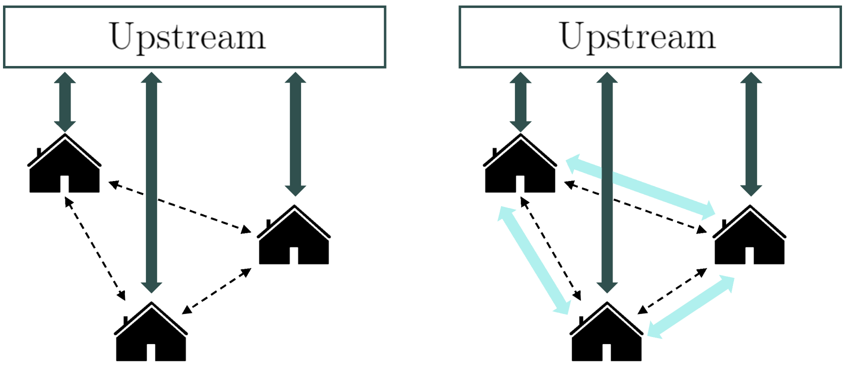


Figure 2.7.: Energy consumption scheduling (left) and energy exchange scheduling (right). Dashed arrows indicate coordination and solid arrows show economic scheduled power exchanges.

An entity that has been attracting a lot of attention lately is the *energy community*. Communities are designed to enable a collective organization mode. However, they do not necessarily deal with demand response programs, which shape the decision scope. They can, for instance, organize a simple local market as discussed in the next section.

2.5. Energy communities: a simple prosumer-centric market or an enabler of collective-centric optimization

An energy community is a catchall concept that is extensively used in the current power systems literature. However, there is a leading trend in reducing the concept into either a simple new market space or a product. Most of the time, they are referred to as *community-based markets*, hence assuming that markets are an in fine goal. We provide in this work an alternative proposition that arguably restores the original meaning by avoiding classic market reasoning.

2.5.1. The market(ing) trend

Earlier in this section, various techniques and mechanisms were addressed. Although load management programs found prime relevance in DSM as imagined and conceived by utilities, the liberalization has disrupted their enforcement framework. In a way, the electricity markets have somewhat relegated them to the past because markets are often believed to be the ultimate solution for solving economic problems most efficiently.

Markets and economic equilibrium are theoretical concepts that originate from political philosophy (neo-classic economy) beliefs, which derived an elegant theory from simple mathematics and unrealistic assumptions for allocating scarce resources [47]–[53]. Hence, under many conditions that are usually not met (no market power, resources homogeneity, perfect information, rationality, etc.), a perfect market can be cleared at a single equilibrium that yields the maximum utility and profit for a market price, thus making them efficient. Nevertheless, only capital and workforce are considered as scarce resources. Markets do not value natural resources (considered as infinite) nor consider negative externalities such as pollution and its consequences [54]. This is why, for instance, oil and gas prices do not increase significantly, whereas their impact is tremendously growing, and their stocks are falling. Only the economic conditions for extracting them and the demand are considered as relevant information. The only workaround that has been found is to create new dedicated markets for trading emissions of pollutants (cf. ETS in

1.1.1). But it is circumscribed to one or a few externalities and is subject to all market caveats as well. It seems, therefore, a hazardous bet to remedy the market inefficiencies by multiplying markets with their own inefficiencies. In addition, ensuring the legibility of such a market bunch is a vain attempt.

Even so, markets are today the main driver of the electricity supply chain. Although they have been in use for a long time already to trade the electricity commodity, electricity markets now aim at giving access to all related actors. In particular, prosumers, who hold today increased control over their energy resources, are expected to play a prime role. They also seek to enable the monetization of any electricity-related service such as balancing mechanisms or the numerous ancillary services. Emerging prosumer-centric markets are envisioned to tackle the great challenges that the electricity sector must face to reduce its environmental footprint. However, beyond the inherent flaws of perfect markets, the hypotheses for market efficiency are even less likely to be met when considering smaller local markets (e.g., the absence of market power requires a large number of small actors to prevent significant markets shares, which is not the case in small entities).

Concurrently, we observe much effort from economic players for making electricity a differentiated product [55], [56] (e.g., wind-produced only energy or locally sourced energy). Rationality and homogeneity are thus excluded. Another game-changer is the increasing capability of storing electricity. Electricity markets were somewhat spared by speculation because of their physical constraints that prevented their accumulation in the form of stocks. Such stocks, as seen for many other commodity markets, could produce undesirable effects.

Altogether, these elements tend to violate the economic equilibrium conditions and leave very little ground for actual efficiency [57]. Although for ideological reasons, markets have been implemented and are now well established for many areas of the economy, a critical subject such as energy, which mobilizes tremendous amounts of resources, probably cannot afford to overlook the many underlying flaws.

2.5.2. Energy communities and collectives

"Think global, act local." This trendy slogan has found extensive application scope for societal issues such as environmental projects, education programs, business cases, etc. The electricity sector is also trying to find its counterpart. As detailed in the previous sections, there are still relatively few players (concentrated markets). While there is still much debate in the sector for visioning the answers to the different economic and environmental challenges, the importance of end-user engagement is unanimously recognized.

End-users until today have been considered a uniform passive group of consumers. The variety of behaviors, consumption profiles, load compositions, and today the distributed energy resources they own represent so many possible degrees of actions. Given this potential and the recent advances in communication and control technology, it is now possible to empower them through innovative solutions. This raises the question of which structure should be adopted. Different models actively involving prosumers compete. In that context, energy communities and collectives occupy a new predominant position.

A *community-based market* (CBM) is an economic entity that implements an internal market functioning. It is included in the broader definition of *prosumer-centric markets*, which enhances end-user engagement through individual preferences and social perspectives. The common denominator is hence discrimination. Individuals can form a community because they share the same opinion about how they should source their energy or because they agree on any other subjective opinion. Other communities are formed because they just share the "rational" goal of optimizing their profit [58]–[63]. *Peer-to-peer markets* are, in that case, an even more flexible solution. Such markets remove all barriers from free trade by allowing all prosumers to engage in direct transactions [64]–[74]. To seize better market opportunities, individuals can even form virtual power plants through peer-to-peer platforms. They are then termed *federated power plants* (FPP) [66]. The common denominator of such a collective is the aggregation of efforts and resources to have a significant enough size on markets. It is, however, difficult to justify the term of community because profit is the only aspiration. Markets hence give the community either a communitarian version, which tends to lose touch with the real issue (adopt a sustainable use of energy) or a pure product of profit-making.

Community-based and peer-to-peer markets both adopt a subordinated view of grid costs, and they represent an important concern for the related actors (TSOs, DSOs, etc.). Only a few exogenous (e.g., transportation like zonal pricing [75]) or very unfair (e.g., based on distribution locational marginal pricing [68]) mechanisms have been proposed. This is because the grid intrinsically limits market opportunities. Grid fees are, therefore, overlooked while it is the main source of expenses in the electricity supply chain. Market approaches thus have a very flexible perimeter and do not need to respect physical boundaries endogenously. However, granting prosumers subjective peer-to-peer exchanges could be physically demanding depending on the incentives. The power network was not designed for arbitrary interaction schemes. The most liberal propositions would require a profound and costly transformation, possibly missing the target of a sustainable energy transition.

In the existing propositions of market approaches, fairness or efficiency are thus jeopardized. This work's proposition is to propose an alterna-

Box 2.5: European legislation on energy collectives

As previously mentioned (cf. 1.1.2), the EU has already defined two official entities that formalize energy communities in its legislation: *renewable energy communities* (RECs) and *citizen energy communities* (CECs). The two are closely related and refer to a way of organizing rather than the definition of specific activities (e.g., promoting renewable self-consumption). There are, however, more stringent criteria for RECs than CECs. RECs must be geographically limited, democratically governed, and eligible participants are fewer [21]. A notable exception affects the effective control of the entity, which is restricted to natural persons, local authorities, or small enterprises for a CEC but also includes medium businesses for RECs. Both RECs and CECs differ from traditional electricity markets because they are owned and operated by their stakeholders for the primary purpose of providing them environmental, economic, or social benefits rather than financial profits. Besides, they are entitled to engage in generation, distribution, supply, consumption, aggregation, energy storage, and other energy services. These community definitions are accompanied by related directives that commit member states to establish local frameworks for their development. However, it could be criticized that the European vision about energy communities is incomplete because the absence of a defined activity could lead to many heterogeneous implementations without any systematic objective. In particular, they do not specifically address the optimized use of energy resources. Moreover, there is no restriction to the definition of community-based markets and other collectives that promote the sole financial profit.

tive addressing these two issues. In particular, it intends to mobilize prosumer engagement through *collective-centric optimization.* The mutualization denominator is used to form *responsible energy communities* that aim to optimize the use of resources. Based on demand-side management and load management tools, energy flexibility and collaboration between members are incentivized by adopting fair distribution schemes. As further exposed, both the commodity and the grid costs are endogenously addressed. This is possible because responsible energy communities are electrically delimited. They can be formed by the prosumers located on one or several distribution feeders or encompass a whole distribution network. The definition of such communities is, hence, tailored to meet local realities and coordinate different solutions to serve a global scheme. Depending on the underlying network structure, the environment complementarity, and the general interactions between its partners, they could correspond to a coherent entity such as a neighborhood, a city, a business, or an industrial park. They hence exploit the current structure of the network and require limited grid investments.

Table 2.1.: Characteristics of energy collectives.

	CBM	FPP	ReEC
Design	market	market	costs
Denominator	differentiation	aggregation	mutualization
Boundaries	economic	none	electrical
Grid allocation	exogeneous	exogeneous	endogeneous

The concept of "Responsible Energy Community" (ReEC) introduced by this work is compared with market designs against several criteria in table 2.1. The next subsection outlines the main characteristics of the ReECs and highlights the key feature adopted in this work: mutualization.

2.5.3. Outline of responsible energy communities

Interactions form the central substance of energy communities. They can be purely economic in nature through energy trading, but they are, more importantly, tangible facts and actions that make the problem binding.

Responsible energy communities are formalized through the federation and aggregation of involved participants under a single electrically delimited entity. That entity optimizes its operations (and investments possibly) and divides costs following a common agreement. The key factor underlying all possible interactions inside a ReEC is *mutualization*.

Mutualization consists of pooling and sharing resources of different natures. In this specific context, we refer to the mutualization of energy resources. We distinguish two types of mutualization grounds:

i) **De facto-shared resources.** They are the energy resources that are shared at all times for a typical operation of power exchanges and supply. The most manifest resource is the electrical network, including power lines, transformers, and substations apparatus. It is the backbone of the supply chain and is considered to perform a public service mission. Such common resources involve high capital and operational expenditures due to their construction, maintenance, and overhaul. Community members interact through these resources in all cases, and they should thus contribute to the expenses according to the use they make.

ii) **Deliberately-shared resources.** They correspond typically to individual excess resources that are pooled, aggregated and shared to produce increased cost efficiency or reliability (e.g., excess photovoltaic production). They are the essence of the responsible energy communities we define because they are not straightforward and require insurance mechanisms to be activated.

Practically, sharing resources either plays directly on the marginal cost an individual can achieve or on the possible action set that is made available. As formalized later in the optimization formulations (chapters 4 through 6), the individual's cost function or the constraints are respectively affected.

The following descriptive (Box 2.6) summarizes the characteristics of the ReECs. They are incrementally implemented in the scenarios detailed in chapters 4 through 6.

Box 2.6: Responsible energy communities - ID card

- **What?** Optimizing the true (non-linear) cost of power exchanges and billing it fairly (aggregation of costs and accounting for flexibility).
- **Who?** A community (perimeter of action) of prosumers (actors) owning time flexible appliances.
- **Where?** A coherent electric network (a low-voltage feeder up to a small medium-voltage grid).
- **When?** Day-ahead horizon (chapters 4 to 6) and investment horizon (Chapter 7).
- **How?** Energy exchange scheduling using time flexibility and mutualization.

2.6. Conclusions

This chapter has illustrated the major paradigm shift happening in the electricity sector and laid the ground for the proposal of this work. Starting from the definition of the different actors and the current organization of traditional electricity markets, it has shown that there is a significant trend towards decentralization.

The leading triggers of decentralization are the electrification of the loads, the growing penetration of distributed generators, the increasing storage capabilities, the development of multicarrier systems, and large-scale digitalization. The consequences are numerous, but the most striking one is the developing active role of end-users. For many years, they were confined to behave as simple consumers, totally disconnected from their environment and the upstream conditions (e.g., weather conditions and network congestions). They are now expected to become a significant flexibility source.

Hence, the instruments enabling the prosumer era, namely demand-side management and the broader notion of load management, were introduced and detailed. A particular focus was brought to energy exchange scheduling. It consists of using the time flexibility one consent for some devices to minimize operation costs (commodity and grid fees). Indeed, whereas residential demand is usually poorly inelastic, many modern devices can be programmed in time.

As a result of these radical changes and the new technical developments in the electricity sector, there has been abundant research on defining new market designs, usually featuring decentralized schemes. Given the neo-liberal economic trend in most countries, cf. Chapter 1, consumer-centric markets have been excessively promoted. Among the main market designs, local replicas of current electricity markets such as community-based markets have been put forward. Concurrently, pushed by the advances in communication and transaction possibilities (e.g., blockchain), peer-to-peer markets have also made their way.

The philosophy behind responsible energy communities is radically different than what community-based markets suggest. Whereas the latter aims at maximizing one arbitrary and simplistic definition of social welfare, the former aims at minimizing cost in the broader sense, i.e., societal and environmental, in a fair way.

Responsible energy communities are, therefore, the alternative we put forth in this work. They have been outlined, and mutualization, its central feature, was highlighted. It was acknowledged that costs are intrinsically non-linear and induce interdependence between participants, thus raising the issue of cost distribution. Reaching an efficient solution satisfying everyone requires resorting to essential mathematical tools. Hence, the next chapter introduces the fundamentals of those tools necessary to apprehend the remainder of this work. In addition, it provides an insight into the characteristics of economic equilibria that will serve as the basis of discussions.

CHAPTER 3.

At the crossroad of equilibrium concepts: economic and mathematical fundamentals

> Too large a proportion of recent "mathematical" economics are merely concoctions, as imprecise as the assumptions they rest on, which allow the author to lose sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols,
>
> *John Maynard Keynes*

3.1. Introduction

In the last chapter, the concept of *responsible energy communities* was introduced. One of their main features is collective-centric optimization through DSM or load management programs. In this work, the energy exchange scheduling problem is more specifically addressed. In particular, the problem is solved on the day-ahead horizon. It allows to derive the next-day operational planning (cf. chapters 4 to 6) but also to evaluate the operational expenditures (OPEX) of the investment planning problem (cf. Chapter 7).

The energy exchange scheduling process, taking place in responsible energy communities, is governed by some generic types of strategic interactions. Mutualization, federation, and aggregation of electrical resources, which are at the core of the scenarios, have practical consequences on their operations and outcomes. These sources of interdependence were identified in the previous chapter to grasp the nature of such communities, cf. 2.5.3.

Energy exchange scheduling, unless it is directed and solved by an external actor, is thus a *multi-agent* problem. Although the optimization of aggregated costs is usually a desirable outcome, prosumers, in practice, try to minimize their costs selfishly. Besides, because there is interdependence between the prosumers' performances through their cost function and possibly their constraints, the individual strategies and outcomes can be characterized by a *non-cooperative game*. In turn, if it is the strategies and outcomes attained by groups of prosumers that are studied (coalitions study), then it falls under *cooperative game theory*. The reverse problem, which aims at designing an allocation to attain a defined outcome is known as *mechanism design*. The combination of these conceptual tools is used to develop and study the various scenarios of energy exchange scheduling in this thesis. Furthermore, computing such a type of problems calls for various other mathematical tools, including *convex optimization*, *complementarity theory*, and *variational inequalities*. An insight on their principles, their mutual relationships, as well as their use in this particular context, are provided in sections 3.2 and 3.3.

Besides, designing community scenarios in a cost-reflective way diverges quite significantly from regular market clearing. The latter adopts a defined market functioning and computes prices (output) in a top-down manner, whereas the former uses costs as input and leads to design scenarios in a bottom-up fashion. However, it is interesting to compare both approaches (cost-based and market-based) in terms of the desirable properties that their equilibria hold. This comparison is proposed in Section 3.4.

3.2. Convex optimization

Convex optimization underlies most of the mathematical reasoning developed throughout this thesis. It is a very broad and rich subject, and this section means to cover only the basic definitions and concepts necessary for the understanding of the developments. Extensive literature is available on the subject [76]–[78], and guided the writing of this section.

Convex optimization problems are a subclass of optimization problems that features a convex objective function and a convex feasible set. In recent years, this computational tool has become of central importance in the engineering and economic fields because it can solve many types of problems using well-developed algorithms. Convex optimization combines disciplines of optimization, convex analysis, and numerical computation.

Power system planning and operation are generally large-scale, non-linear problems that are not easily tractable. Convex optimization offers multitudinous opportunities for relaxing such problems efficiently, in particular conic programming, which is further detailed.

3.2.1. Basic definitions

A large variety of real-world problems can be presented under the generic form of a constrained optimization problem.

Definition 3.1. A *constrained optimization problem*, in its standard form, is written as

$$\begin{aligned} \min_{x} \quad & f_0(x) \\ \text{s.t.} \quad & f_i(x) \leq 0, \quad i = 1, \dots, m \\ & h_i(x) = 0, \quad i = 1, \dots, p. \end{aligned} \tag{3.1}$$

where $x \in \mathbb{R}^n$ is a vector of decision variables, $f_0(x)$ is the objective (or cost) function to be minimized and the $f_i(x)$ and $h_i(x)$ are respectively the inequality and equality constraints.

In the following definitions and concepts, we refer extensively to convex sets and functions. We recall their definitions.

Definition 3.2. A *convex set* is a set $\Omega \subseteq \mathbb{R}^n$ for which any two points, $x, x' \in \Omega$ are joined by a segment that belongs to Ω

$$\alpha x + (1 - \alpha)x' \in \Omega, \quad \forall x, x' \in \Omega \text{ and } \alpha \in [0, 1]. \tag{3.2}$$

Examples of non-convex and convex sets are illustrated on Figure 3.1.

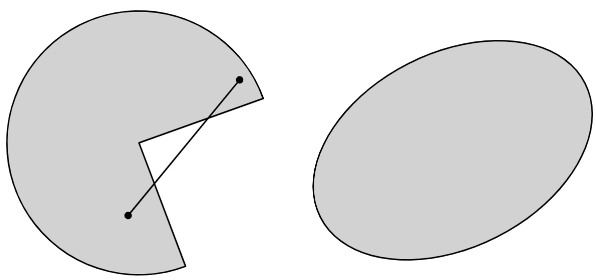


Figure 3.1.: A non-convex set (left) and a convex set (right) in two dimensional space. A set for which any segment formed by two points is not entirely included in that set is non-convex.

Definition 3.3. A *convex function* is a function $f(x) : \Omega \to \mathbb{R}$ if $\forall x, x' \in \Omega$ where $\Omega \subseteq \mathbb{R}^n$ and $\alpha \in [0, 1]$

$$f(\alpha x + (1-\alpha)x') \leq \alpha f(x) + (1-\alpha)f(x'). \tag{3.3}$$

An equivalent statement is to condition that the epigraph of the function, i.e., the subspace of Ω_{n+1} containing all points above the graph of the function, is a convex set. These properties are depicted in Figure 3.2.

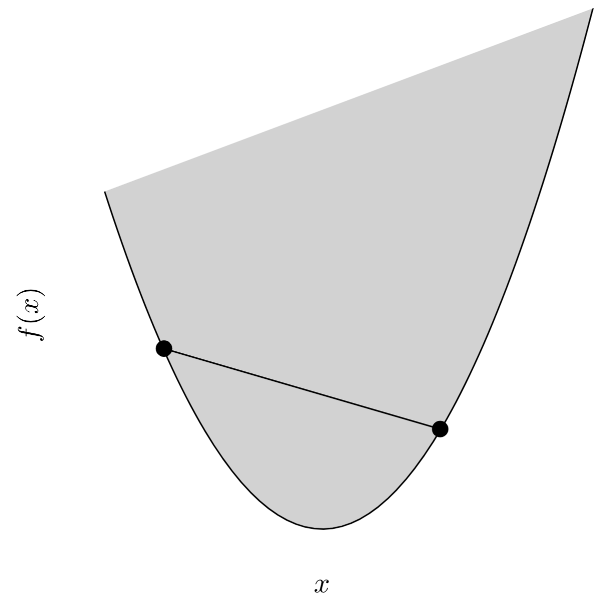


Figure 3.2.: Any segment joining two points of the graph must be in the epigraph, which must be a convex set.

Using the above definitions, it is possible to define a very important subclass of optimization problems.

Definition 3.4. A *convex optimization problem* is an optimization problem such as in (3.1), for which $f_0(x)$ and the $f_i(x)$ are all convex and the $h_i(x)$ are affine.

Convex optimization problems are ensured not to have local optima, which is a very attractive property. There are powerful analytical and algorithmic tools available to study these problems that make them highly tractable. Besides, a large number of applications can be formulated in this way with crisp relaxation.

3.2.2. Optimality conditions

Optimality conditions constitute the theoretical foundations for the design (e.g., stopping criterion) and analysis (e.g., convergence) of optimization algorithms.

They are also appropriate for further analyzes, such as sensitivity analysis.

The central optimality condition in optimization is called the *minimum principle*. It is formally defined as follows.

Theorem 3.1. *Consider an optimization problem such as in* (3.1) *where* f_0 *is differentiable and the feasible set* Ω *is convex.*

- *If* $x^\star$ *is a local minimum of* f_0 *over* Ω*, then*

$$(x - x^\star)^T \nabla f_0(x^\star) \geq 0, \quad \forall x \in \Omega. \tag{3.4}$$

- *If* f_0 *is convex, then the above condition is sufficient for* $x^\star$ *to be the global minimum.*

A geometric interpretation of the minimum principle is provided on Fig. 3.3.

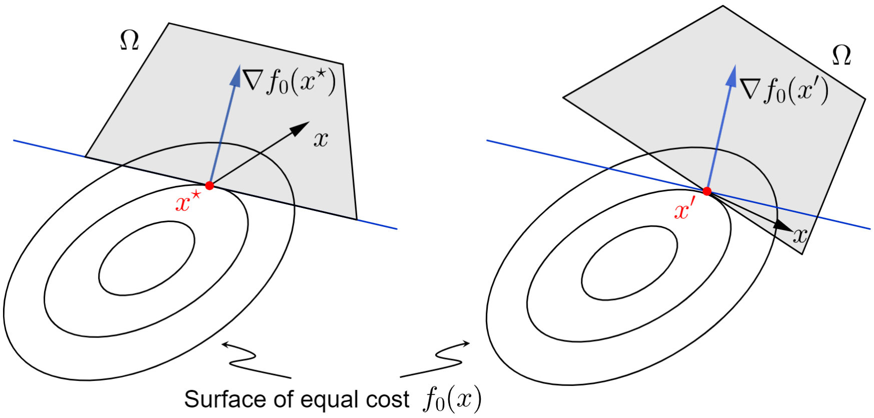


Figure 3.3.: A feasible point $x^\star$ that satisfies the minimum principle (left) and feasible point x' that does not (right) as all vectors $x - x'$ do not make a non-obtuse angle with $\nabla f_0(x')$.

For constrained sets Ω defined by equalities and inequalities, the minimum principle can be enhanced under other optimality conditions: the Karush-Kuhn-Tucker (KKT) conditions.

Theorem 3.2. *Let* $\bar{x}$ *be a feasible solution of an optimization problem* (3.1) *and let* $\mathcal{I} = \{i \mid f_i(\bar{x}) = 0\}$*. Suppose that* $\nabla f_i(\bar{x})$ *for* $i \in \mathcal{I}$ *and* $h_i(\bar{x})$ *for* $i = 1, \ldots, p$ *are linearly independent. If* $\bar{x}$ *is a local minimum, then there exists unique Lagrange multipliers* (λ, μ)*, such that*

$$\begin{aligned}\nabla f_0(\bar{x}) + \sum_{i=1}^{m} \lambda_i \nabla f_i(\bar{x}) + \sum_{i=1}^{p} \mu_i \nabla h_i(\bar{x}) &= 0 \\ \lambda \succeq 0, \quad \lambda_i f_i(\bar{x}) &= 0 \quad \forall i = 1, \dots, m \\ f_i(\bar{x}) \leq 0 \quad \forall i = 1, \dots, m, \quad h_i(\bar{x}) &= 0 \quad \forall i = 1, \dots, p.\end{aligned} \tag{3.5}$$

If the optimization problem is convex, then $\bar{x}$ is a global optimal solution.

The KKTs form a system of equations and inequalities (constituting a mixed complementarity problem, cf. Box 3.2) that can be solved in some specific cases to derive a closed-form analytical solution. In general, they are used to analyze the solutions and constitute the basis of numerical methods for solving optimization problems. We refer to [76] for the details.

3.2.3. Classification and algorithms

In order to introduce some of the canonical problems we mention further, we provide the following overview of convex problem classes.

Conic optimization is a subfield of convex optimization that offers convenient analyzes of fundamental concepts. Optimization problems that can be formulated in such a form are therefore highly tractable. The main types of conic programs are the following:

i) **Linear programming:** it consists of optimizing a linear function on a convex polyhedron:

$$\begin{aligned}\min_x \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b,\end{aligned} \tag{3.6}$$

where $A \in \mathbb{R}^{m \times n}$ is a matrix and $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$ are vectors. Linear optimization problems encompass most application fields and sectors. In the electricity sector, a linear approximation of the optimal power flow problem, commonly referred to as DC OPF, is a notorious example. Another prominent example is economic dispatch. The main benefit to expressing a problem as a linear program is that they are highly tractable. Nonetheless, they usually arise from the simplification of the original problem at the cost of accuracy. Originally, it was the first type of problem that was addressed. The two most well-known solution algorithms are the *simplex* algorithm and the *interior-point method.*

ii) **Quadratic programming:** they include a broader range of problems than linear problems because their objective function can be quadratic. It is hence of the form

$$\begin{array}{ll} \min\limits_{x} & \frac{1}{2}x^TQx + c^Tx \\ \text{s.t.} & Ax \leq b, \end{array} \tag{3.7}$$

where $Q \in \mathbb{R}^{n\times n}$ is a symmetric matrix, $A \in \mathbb{R}^{m\times n}$ is a matrix and $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$ are vectors. Most solvers use an extension of the simplex algorithm or an extension of the interior-point method.

iii) **Second-order cone programming:** it is a convex optimization problem minimizing a linear function over the intersection of an affine set and the product of second-order cones, cf. 3.4.

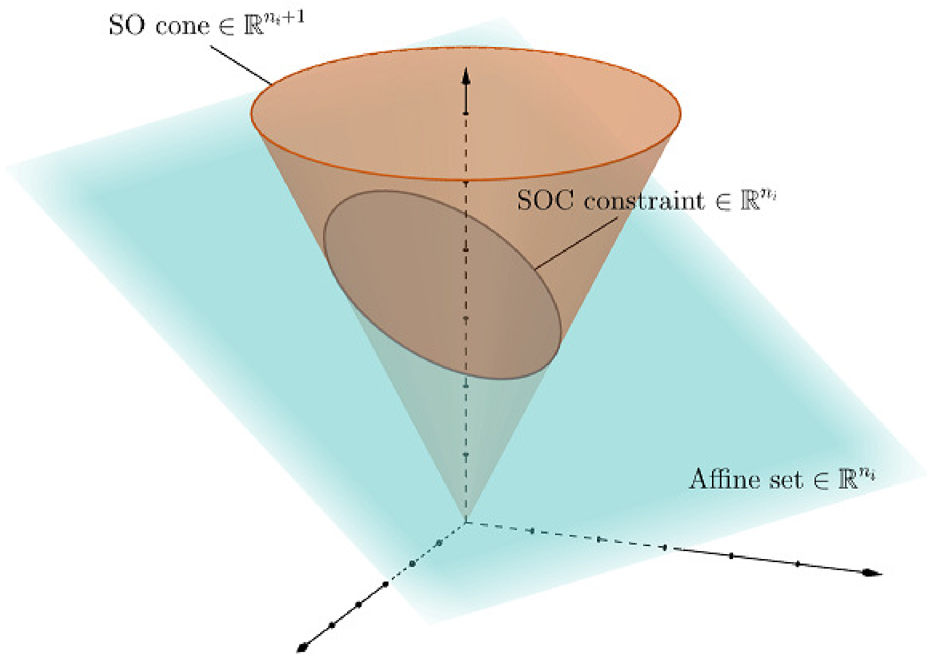


Figure 3.4.: A SOC constraint corresponds to the intersection of the plane formed by the affine set of constraints and the second-order cone.

It is formulated as

$$\begin{array}{ll} \min\limits_{x} & c^Tx \\ \text{s.t.} & ||A_ix + b_i||_2 \leq c_i^Tx + d_i, \quad i = 1, \dots, N, \end{array} \tag{3.8}$$

where $c \in \mathbb{R}^n$, $A_i \in \mathbb{R}^{(n_i)\times n}$, $b_i \in \mathbb{R}^{n_i}$, $c_i \in \mathbb{R}^n$, and $d_i \in \mathbb{R}$ are the parameters. The second order cone constraints arise from the $\mathbb{R}^{n_i+1}$ cones. Many engineering problems can be recast as second-order cone programs. In this way, optimal power flow problems can be reduced to such a form without too much approximation (cf. 6.2.2). Interior point methods are usually efficient for solving SOCPs.

iv) **Semi-definite programming:** they are a much more general class of convex optimization problems that consists of minimizing a linear function over the intersection of the cone of positive semi-definite matrices with an affine space:

$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{s.t.} \quad & F(x) \succeq 0, \end{aligned} \tag{3.9}$$

where

$$F(x) \triangleq F_0 + \sum_{i=1}^{m} x_i F_i,$$

and where $F_0, \ldots, F_m \in \mathbb{R}^{n \times n}$ are symmetric matrices, and $c \in \mathbb{R}^m$ a vector. Such problems are not much harder to solve, and most interior-point methods have been generalized for this class of problems.

Figure 3.5 summarizes the convex optimization problem classes and illustrates their hierarchy.

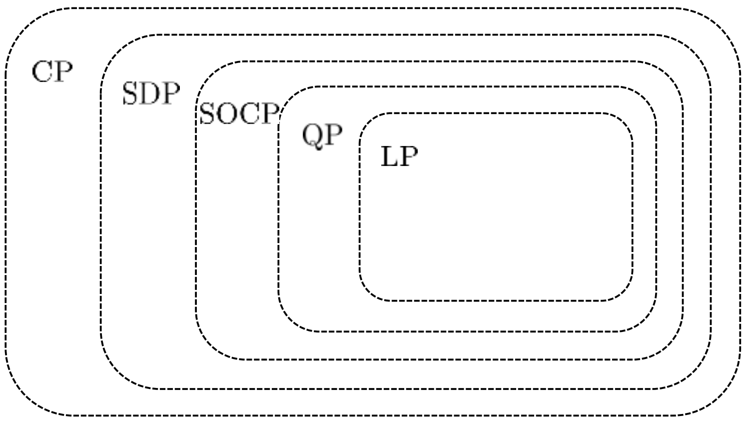


Figure 3.5.: A hierarchy of convex optimization problems. (CP: conic program, SDP: semi-definite program, SOCP: second-order cone program, QP: quadratic program, LP: linear program)

3.3. Game theory and Mechanism Design

As previously introduced, the energy exchange scheduling of responsible energy communities can be formalized as a game. Game theory is a mathematical field that studies a system of self-interested agents in conditions of strategic interactions [79]–[81]. It is widely popular today and has numerous applications ranging from mathematics to economics, sociology, biology, and data science.

One central notion to examine a game-theoretical problem is utility theory. Utility quantifies an agent's preference given each outcome. It finds its roots in utilitarianism philosophy [82], which describes actions that maximize the happiness and well-being of individuals. The utility function of an individual maps a state of the world into a real number.

Although players are the common entity for all games, it is usual to distinguish between non-cooperative and cooperative game theory. These terms can be misleading because both theories can refer to individuals that have conflicting or non-conflicting interests. In non-cooperative game theory, the model primitives are the possible actions of individual players whereas, in cooperative game theory (a.k.a. coalitional game theory), the possible joint actions form the primitives.

The remainder of this section does not mean to introduce or summarize any of the mentioned theories, but rather to highlight a collection of the most important notions that affect this work.

3.3.1. Strategic games and solution concepts

Non-cooperative games study the player's set of possible actions and their preferences over the possible joint action set, where each player chooses its strategy autonomously. The most common non-cooperative game is the strategic game.

In a strategic (or normal-form) game [80], players choose their action plan at a single and same time step such that they cannot respond to another's selection. Such representation of a game is fundamental because most other representations (extensive forms) can be reduced in that form.

Definition 3.5. Formally, a *strategic game* is defined by the tuple $\langle \mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}} \rangle$ where:

- $\mathcal{N}$ is a finite set of N players.
- $\mathcal{S}_i$ is player i's strategy (possibly infinite) set. The vector of strategies $s = (s_1, \ldots, s_n) \in \prod_{i\in\mathcal{N}} \mathcal{S}_i$ is referred to as the strategy profile.

- $u_i : \prod_{i\in\mathcal{N}} \mathcal{S}_i \to \mathbb{R}$ is player i's real valued utility (or payoff) function.

Note that strategies can either consist of selecting and playing a single action or selecting different actions and playing one of them randomly according to probability distributions. They can therefore be different from one play to the other. The former are named *pure strategies* and the latter *mixed strategies*.

If the strategy set of a player is finite, the most straightforward representation of a strategic game is a N-dimensional matrix of payoffs (cf. Box 3.1). However, if the strategy set is not finite (continuous game), it is not possible to derive such matrices.

Game theory finds its interest mostly in identifying appealing strategies, which are called solution concepts.

A widely used solution concept consists of defining the most desirable outcomes from an external viewpoint. The most widely used solution concept in such context is Pareto efficiency.

Definition 3.6. A strategy profile s is *Pareto efficient* if there is no other strategy s' such that:

- $\forall i \in \mathcal{N}, u_i(s') \leq u_i(s)$;
- $\exists j \in \mathcal{N}, u_j(s') < u_j(s)$.

In other words, Pareto efficient strategy profiles are solutions for which no player can be better off without decreasing another's utility.

Definition 3.7. *Social welfare*, in the context of game theory (it has a different definition in market theory), corresponds to the sum of the utilities: $\sum_{i\in\mathcal{N}} u_i(s)$.

Definition 3.8. A strategy profile s is the *social optimum* if the social welfare is maximized.

If we consider the game from an individual point of view and assume that all players act rationally (i.e., they maximize their utility), the agents have to determine their best response. Let s_{-i} denote the set of all strategy profiles except those of player i and $\mathcal{S}_{-i}$ their corresponding strategy set.

Definition 3.9. The strategy s_i is a *best response* of player i against $s_{-i} \in \mathcal{S}_{-i}$ if $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}), \forall s'_i \in \mathcal{S}_i$. We denote the set of best responses against s_{-i} by $\mathcal{B}(s_{-i})$.

The most notorious solution concept of non-cooperative game theory is the Nash equilibrium (named after the mathematician John Forbes Nash [83]).

Box 3.1: An illustrative example: the prisoner's dilemma

The prisoner's dilemma is one of the most famous examples of a strategic game. Although the energy exchange scheduling games that are further developed cannot be represented under their matrix form because they are continuous games, the matrix representation allows a simple analysis without any loss of generality. In this example, two players (prisoners) can adopt two different strategies (perform one of two different actions). Either they "cooperate" (C) or "defect" (D). The payoff matrix is the following:

		Player 2: C	Player 2: D
Player 1	C	2 , 2	0 , 3
	D	3 , 0	1 , 1

Table 3.1.: Payoffs matrix of the prisoner's dilemma.

There is one social optimum. It corresponds to (C;C), i.e., both prisoners cooperate. Indeed, their payoffs sum up to 4. There are three Pareto efficient strategy profiles: (C;C), (C;D), and (D;C). Indeed, it is not possible when these profiles are adopted to improve one's position without deteriorating the payoff of the other one. Finally, there is one Nash equilibrium which is both players defecting (D;D). In that case, none of the two prisoners can regret their choice.

Definition 3.10. The *Nash equilibrium* is a strategy profile s for which, for all agents $i \in \mathcal{N}$, s_i is a best response to s_{-i}. The Nash equilibrium is usually denoted by $s^\star$, such that $u_i(s_i, s^\star_{-i}) \leq u_i(s^\star)$ holds.

The intuition is that no agent can unilaterally change its strategy while improving its outcome. The Nash equilibrium is often described as a no-regret equilibrium because players choose their best possible play depending on the predictions of others' play. Note that the Nash equilibrium can correspond to pure or mixed strategies. They are referred to as pure or mixed Nash equilibria.

Finding the Nash equilibrium is usually a tedious task. First, it should be determined if the game features one. One of the major result that John Nash proved is:

Theorem 3.3. *All finite games with a finite number of players and strategy space have at least one (mixed) Nash equilibrium.*

It is based on Kakutani's fixed point theorem, cf. [84].

Another result based on the works of Nash, Debreu, and Fan is extensively used in this work.

Theorem 3.4. *Games possess a pure strategy Nash equilibrium if:*

1. *the strategy spaces are nonempty, convex and compact, and,*
2. *players have continuous and quasi-concave payoff functions.*

The problem of solving an arbitrary Nash equilibrium is considered to be complex. It belongs to the PPAD (Polynomial Parity Arguments on Directed graphs) complexity class for which no efficient solution procedure is believed to exist [85]. More insight to its computation is provided in the following subsection.

Depending on the scenarios, it is interesting to formulate the energy exchange scheduling as a non-cooperative game and thus derive appealing results such as the Nash equilibrium or Pareto efficient solutions. It is especially true because individuals have conflicting interests.

3.3.2. Nash and generalized Nash equilibrium problems

Finding the Nash equilibrium of a problem formulated as a strategic game is increasingly popular among decision-makers. When facing situations in which the interactions between agents are not negligible, it can prove to be a better tool than centralized approaches such as global optimization. Depending on the nature of the interactions, the problems can be formulated as one of the following problems.

i) **Nash equilibrium problem (NEP).** The interactions take place at the level of their utility function only (we will use the term objective function in the remainder of this work, which is closer to the optimization world vocabulary).

ii) **Generalized Nash equilibrium problem (GNEP).** Beyond the interactions that take place at the level of objective functions, the actions of other agents influence one individual's feasible strategy set. Such problems are much more complicated to study and harder to solve.

Nash equilibrium problems consist of a set of coupled optimization problems. Let's consider $\mathcal{N}$, a set of N players. Each player $n \in \mathcal{N}$ competes against the others by choosing its strategy x_n to minimize its objective function $b_n(x)$. Note that to ease the comparison with the developments of this thesis, the minimization of the objective function is considered to maximize the utility of an individual, i.e., $b_n(x) = -u_n(x)$ is meant to be minimized. Following the same idea, the notation is adapted.

Let $x_{-n} \triangleq \{x_m\}_{m\in\mathcal{N}\setminus\{n\}}$ denote the set of all the strategies (strategy profile) except those of player n. We formally define such a game by the tuple $\mathcal{G} \triangleq \langle \mathcal{N}, \Omega, \mathbf{b}\rangle$, where $\Omega \triangleq \prod_{n\in\mathcal{N}} \Omega_n$ is the joint strategy set, with Ω_n being the individual strategy set of prosumer n, and $\mathbf{b} \triangleq \big(b_n(x_n, x_{-n})\big)_{n\in\mathcal{N}}$ is the vector of all the objective functions.

Definition 3.11. The *Nash equilibrium problem* is a strategic game in which each player n aims at solving the following optimization problem, given x_{-n}:

$$\begin{array}{ll} \min\limits_{x_n} & b_n(x_n, x_{-n}) \\ \text{s.t.} & x_n \in \Omega_n \end{array} \quad \forall n \in \mathcal{N}. \tag{3.10}$$

The solution of a game is the Nash equilibrium, which is a feasible strategy profile $x^\star \triangleq \{x_n^\star\}_{n\in\mathcal{N}}$ with the property that no single player n can benefit by unilaterally deviating from $x^\star$ if all the other players act according to $x_{-n}^\star \triangleq \{x_m^\star\}_{m\in\mathcal{N}\setminus\{n\}}$.

In general, deriving the properties of Nash equilibrium problems such as existence, uniqueness, or convergence of algorithms is complicated (cf. [86]). For simple problems, it is possible to reason from the fact that the Nash equilibrium is a fixed point of the best response mapping. Indeed, if $\mathcal{B}(x) \triangleq \mathcal{B}_1(x_{-1}) \times \mathcal{B}_2(x_{-2}) \times \ldots \times \mathcal{B}_N(x_{-N})$, then a strategy profile $s^\star$ is a Nash equilibrium if and only if it is a fixed point of $\mathcal{B}(x)$, i.e., $x^\star \in \mathcal{B}(x^\star)$. However, its applicability is often limited because it can be difficult to compute the best response mapping $\mathcal{B}(x)$ in closed form. There exists, however, two alternatives:

i) Exploiting the particular structures that some games hold such as potential games or supermodular games [87], [88].

ii) Reducing strategic games to variational inequalities. Variational inequality theory is a well-developed field that provides interesting ground for deriving results about the properties of the solution. It is also useful because of its numerous algorithms to compute solutions. More insight is provided later in 3.3.3.

The generalized Nash equilibrium problem expands the Nash equilibrium problem by adding interdependence between each player's strategy sets, i.e., $x_n \in \Omega_n(x_{-n})$.

Definition 3.12. The *generalized Nash equilibrium problem*(GNEP) is a strategic game in which each player n aims at choosing a strategy $x_n \in \Omega_n(x_{-n})$ that solves the problem:

$$\begin{array}{ll} \min\limits_{x_n} & b_n(x_n, x_{-n}) \\ \text{s.t.} & x_n \in \Omega_n(x_{-n}) \end{array} \quad \forall n \in \mathcal{N}. \tag{3.11}$$

The solution of such a game is called a generalized Nash equilibrium. It has the same properties of the Nash equilibrium but applies for all $x_n \in \Omega_n(x^\star_{-n})$.
Generalized Nash equilibria are becoming widely popular because they naturally arise in the modeling of many practical applications such as energy markets [89]. However, in their general form, they are almost intractable.

One particular class of GNEPs with the substantial ground for results is the so-called generalized Nash equilibrium with shared constraints. It finds many important applications in many domains.

Definition 3.13. *Generalized Nash equilibrium with shared constraints* is a GNEP for which the feasible strategy sets $\Omega_n(x_{-n})$ are defined as

$$\Omega_n(x_{-n}) \triangleq \{x_n \in \mathcal{W}_n : g(x_n, x_{-n}) \leq 0\}, \tag{3.12}$$

where $\mathcal{W}_n$ is the set of individual constraints of n and $g(x_n, x_{-n})$ represents the set of shared coupling constraints.

3.3.3. Variational inequality reformulation of (G)NEPs

Variational inequalities is a general framework encompassing a wide class of problems of mathematics and applied sciences [90], [91]. It is formally described as the following.

Definition 3.14. The *variational inequality problem*, denoted by VI(Ω, $\mathbf{F}$), is to determine a vector $x^\star \in \Omega \subseteq \mathbb{R}^n$, such that:

$$(x - x^\star)^T \mathbf{F}(x^\star) \geq 0, \forall x \in \Omega, \tag{3.13}$$

where Ω is a given closed and convex set, and $\mathbf{F} : \Omega \to \mathbb{R}^n$ is a continuous mapping.

It is interesting to consider the variational inequality problem as a generalization of the minimum principle (3.4) where a general function $\mathbf{F}$ replaces ∇f. Figure 3.6 illustrates a geometrical interpretation of (3.13).
There are thus strong ties with convex optimization, but VIs encompass a much wider range of problems because they are not restricted to functions that are the gradient of another potential function. Hence, variational inequality theory covers classical problems such as

i) non-linear complementarity problems;

ii) mixed non-linear complementarity problems;

iii) system of equations.

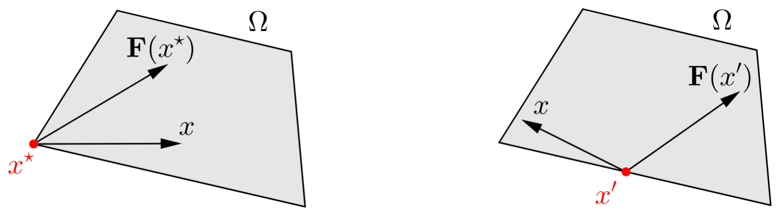


Figure 3.6.: A feasible point $x^\star$ that is a solution of the VI$(\Omega, \mathbf{F})$ (left) because all feasible vectors $x - x^\star$ form an acute angle with $\mathbf{F}(\mathbf{x}^\star)$ and a feasible point x' that is not a solution (right).

Under some conditions, NEPs can be characterized by formulating a suitable variational inequality problem. Indeed, it is shown that the variational inequality problem can be equivalent to a convex optimization problem. Following the minimum principle (3.4), each problem n of a game $\mathcal{G}$ such as in (3.10) must have a solution $x_n^\star$ that satisfies $(x_n - x_n^\star)^T \nabla_{x_n} b_n(x_n^\star, x_{-n}^\star) \geq 0$, for all $x_n \in \Omega_n$. It is thus straightforward to derive the following equivalence.

Lemma 3.1. *If we assume for the game $\mathcal{G} \triangleq \langle \mathcal{N}, \Omega, \mathbf{b} \rangle$ that each player n*

1. *has a closed and convex strategy set Ω_i;*
2. *has a payoff function $b_n(x_n, x_{-n})$ that is continuously differentiable in x and convex in x_n for every fixed x_{-n}.*

Then, the game $\mathcal{G}$ is equivalent to the VI$(\Omega, \mathbf{F})$, where $\mathbf{F}(x) \triangleq (\nabla_{x_n} b_n(x))_{n=1}^N$.

This result is substantial because it is possible to characterize the solution of a Nash equilibrium and use proven algorithms by resorting to the extensive variational inequality theory.

In cases where the feasible sets are dependent on the other players, i.e., GNEPs, connections with variational inequality theory can also be valuable. Such a problem can be formulated as a quasi-variational inequality problem.

Definition 3.15. The *quasi-variational inequality problem*, denoted by QVI$(\Omega$, $\mathbf{F})$ is to determine a vector $x^\star \in \Omega(x^\star) \subseteq \mathbb{R}^n$, such that:

$$(x - x^\star)^T \mathbf{F}(x^\star) \geq 0, \forall x \in \Omega(x^\star), \tag{3.14}$$

where $\Omega(x^\star)$ is a given closed and convex set and $\mathbf{F} : \Omega \to \mathbb{R}^n$ is a continuous mapping.

Unfortunately, to this day, there are still very limited possibilities for solving such a generic problem in this way. However, the specific problem of GNEPs with shared constraints can be solved using a suitable VI problem. Indeed, such a GNEP can be reduced to a standard VI(Ω, $\mathbf{F}$), which yields what are termed variational (or normalized) equilibria. These are necessarily solutions of the GNEP but the reverse, i.e., GNEP equilibria that are a solution of the associated VI, is generally not true.

The Variational equilibrium has the interesting characteristic that all the Lagrange multipliers associated with the global constraints are equal among players, i.e., $\lambda_1 = \ldots = \lambda_N$ [96]. They are hence often considered as a fair treatment solution towards the constraints. Moreover, its uniqueness is guaranteed under proper conditions, and the VI formalization allows the straightforward derivation of distributed algorithms.

3.3.4. Coalitional games

As previously introduced, coalitional (or cooperative) game theory is a branch of game theory that assumes the group as the elementary modeling unit instead of individual agents. Unlike non-cooperative game theory, it is interested only in the preferences (translated mathematically by the utility or payoff) of individuals rather than their actions. The outcome of such a game is the coalition that will form and its joint set of actions.

Most commonly, a coalitional game assumes that there is *transferable utility* among players of the coalition. This assumption is satisfied when a single payoff value can be assigned to a coalition. In other words, players of the coalition share the same type of utility, and there is no restriction on how it is divided among them. The most straightforward utility is money. If there is non-transferable utility (e.g. some players prefer to minimize their bill while others prefer to maximize autoconsumption), payoffs are computed individually, cf. [97].

Definition 3.16. A *coalitional game with transferable payoff* consists of a pair $\langle \mathcal{N}, v \rangle$, where

- $\mathcal{N}$ is a finite set (of players), also called the grand coalition.
- v is a function that associates with each non-empty coalition $\mathcal{S} \subseteq \mathcal{N}$ a real number $v(\mathcal{S})$ which is called payoff or the worth of a payoff.

An interesting and useful property that is often assumed for coalitional games is superadditivity.

Definition 3.17. A game $\mathcal{G}\langle \mathcal{N}, v \rangle$ is *superadditive* if for any subsets $\mathcal{S}, \mathcal{T}$

$$v(\mathcal{S}) + v(\mathcal{T}) \leq v(\mathcal{S} \cup \mathcal{T}), \quad \forall \mathcal{S}, \mathcal{T} \subseteq \mathcal{N}, \mathcal{S} \cap \mathcal{T} = \emptyset.$$

This means that the value of combined disjoint coalitions is always equal or greater than the sum of these separate coalitions.

A key question of coalitional game theory is how the payoff should be distributed among participants of the grand coalition. The reason is twofold. Firstly, the superadditivity, commonly assumed for such games, yields the highest value for the grand coalition. Secondly, in many application cases, participants are bound to the grand coalition for legal and physical reasons (e.g., participants share the same electricity distribution network).

In order to analyze coalitional games, the following terminology is introduced.

Definition 3.18. Let the mapping $\phi : \mathbb{N} \times \mathbb{R}^{2^N} \mapsto \mathbb{R}^N$ from a coalitional game to a vector of N values denote the *payoff distribution* and $\phi_i(\mathcal{N}, v)$ denote the i^{th} value.

We refer to the latter by x_i as a short form when there is no possible confusion.

Besides, some definitions characterizing the payoff distribution should be provided. A basic definition is the imputation.

Definition 3.19. *An imputation* of the coalition game $\langle \mathcal{N}, v \rangle$ is a feasible payoff profile x for which $\forall i \in \mathcal{N}, x_i \geq v(i)$.

This is also known as *individual rationality*, i.e., the individual obtains at least as much as it could obtain on its own, without cooperating with anyone else. Note that depending on the authors, imputations may require to feature an *efficient* payoff profile, i.e., $\sum_i^N x_i = v(\mathcal{N})$. We consider this additional requirement in the remainder.

An essential property that is often sought for a game is stability. A game is stable if no deviation is profitable, i.e., no player prefers to form a smaller subcoalition. A strong notion that involves stability is the core.

Definition 3.20. *The core* of the coalitional game with transferable payoff $\langle \mathcal{N}, v \rangle$ is the set of feasible payoff profiles for which

$$\forall \mathcal{S} \subseteq \mathcal{N}, \sum_{i=1}^{N} x_i \geq v(\mathcal{S}).$$

In other words, the core gives a set of profiles (payoff vectors) such that no sub-coalition can offer a greater payoff for any agent.

There is a clear analogy between the core of a coalitional game and the Nash equilibrium of a non-cooperative game. However, whereas the Nash equilibrium entails stability concerning individual deviations, the core implies stability with respect to deviations of any arbitrary coalitions.

In addition to stability, the other most sought property when distributing a payoff is fairness. As highlighted in the introduction, fairness is an abstract and arguable notion that could be discussed and put into perspective with regard to the context. However, there exits a payoff distribution (imputation) that meets three interesting axioms:

Axiom 3.1. *Symmetry.* If $v(\mathcal{S} \cup \{i\}) = v(\mathcal{S} \cup \{j\}), \forall \mathcal{S} \subseteq \mathcal{N}, \mathcal{S} \cap \{i, j\} = \emptyset$, then $\phi_i(\mathcal{N}, v) = \phi_j(\mathcal{N}, v)$.

Axiom 3.2. *Dummy player.* If $v(\mathcal{S}) = v(\mathcal{S} \cup \{i\}), \forall \mathcal{S} \subseteq \mathcal{N}, \mathcal{S} \cap \{i\} = \emptyset$, then $\phi_i(\mathcal{N}, v) = 0$.

Axiom 3.3. *Additivity.* If v and u are payoff functions, then $\phi_i(\mathcal{N}, v + u) = \phi_i(\mathcal{N}, v) + \phi_i(\mathcal{N}, u), \forall i \in \mathcal{N}$.

The payoff function satisfying the above axioms is called the *Shapley value* and is defined as follows.

Definition 3.21. Given the coalitional game $\langle \mathcal{N}, v \rangle$, the Shapley value is given by

$$\phi_i(\mathcal{N}, v) = \sum_{S \subseteq \mathcal{N} \backslash \{i\}} \frac{(|\mathcal{S} - 1|!(N - |\mathcal{S}|)!)}{N!} [v(\mathcal{S}) - v(\mathcal{S} \backslash \{i\})] \tag{3.15}$$

It is also an imputation and therefore meets efficiency and individual rationality.

The interpretation of (3.15) is that the Shapley value is the average marginal contribution of a player over all the subcoalitions involving that player.

3.3.5. Mechanism design

Mechanism design is a subfield of game theory that aims at designing good system-wide solutions in a strategic setting with rational and self-interested agents that may have private information [98], [99]. It is often referred to as reverse game theory because it adopts an objective-first approach. A *mechanism* is modeling a governing interaction of an institution (e.g., a market). In recent years, mechanism design has found many applications in politics and economics, notably in market design and auction theory.

A consistent branch of mechanism design looks at the notion of dominant strategies.

Definition 3.22. Strategy x_n is a *dominant strategy* if it maximizes the agent's expected utility for all possible strategies of other agents,

$$u_n(s_n, s_{-n}) \geq u_n(s'_n, s_{-n}) \forall s'_n \neq s_n, s_{-n} \in \Omega_{-n}. \tag{3.16}$$

Dominant-strategy equilibrium is a stronger solution concept than the Nash equilibrium. It makes no assumptions about how an agent perceives the strategies of other agents, and it does not require any specific prior information.

In this work, we are mainly interested in the properties of mechanisms. Indeed, mechanism design defines desirable properties, which are of topical use in market theory. These considerations are developed in the next section.

3.4. Characterization of economic equilibria

The market economy is the most widespread economic system nowadays. In such a system, supply and demand define prices (market prices), which in turn guide production and consumption. By clearing the market price the surpluses (cf. Figure 3.8) of the offer and the demand, a.k.a social welfare, is maximized.

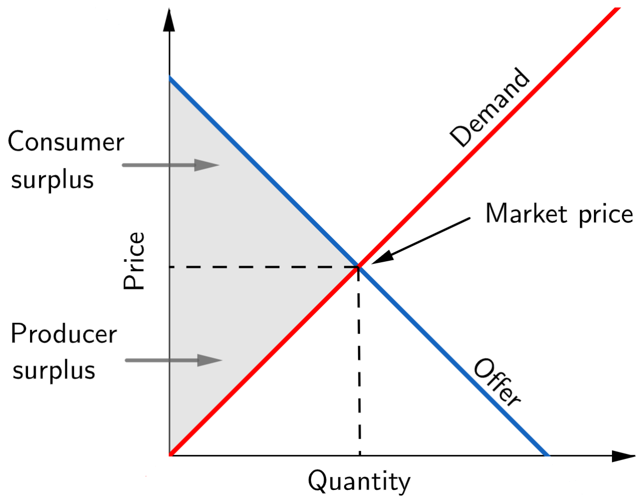


Figure 3.8.: Supply and demand curves meet at the market price. The area in gray between the two curves corresponds to the total surplus.

As already introduced in 2.3, the market logic has been adopted at a large scale for electricity markets. The electricity markets already involve most large producers, suppliers, and consumers. Recently, there has been a trend to organize new prosumer markets that further include small individual production units and flexible resources. This is particularly topical in the context of emerging energy communities.

However, the elasticity of the demand at the low-voltage level is limited, notably for residential consumers. Besides, as previously highlighted, perfect market assumptions are even more tenuous in such a setting. In practice, low-voltage end-users seek to minimize their bill for a given amount of energy needed. Although they do not generate demand level elasticity, they can offer scheduling flexibility. As introduced in 2.5.2, the responsible model of energy communities that is aimed in this work means to optimize the truth costs. These costs derive from an interdependent process of scheduling and exchanges between end-users through the grid.

Both the organized market designs and the cost-based billing designs involve collections of interrelated optimization problems. They thus lead to equilibrium problems. The underlying economic equilibria usually feature one or more desirable properties. Generally, the most sought properties for designs are efficiency, incentive compatibility, individual rationality, and budget balance. They are detailed in the following subsection. Then, the remaining subsections characterize both design approaches according to these properties.

3.4.1. Desirable properties

As previously introduced (cf. 3.3.5), mechanism design aims at achieving a particular outcome by designing economic mechanisms and incentives. It is thus an attractive tool for conceiving markets and billing mechanisms. The theory of mechanism design also defines desirable economic properties [99]. They are the following.

i) **Allocative efficiency.** It refers to the maximization of the total social welfare. As extensively discussed in 2.5.1, it is the cornerstone of market equilibrium theory.

ii) **Incentive compatibility**. An incentive-compatible market relies on an appealing mechanism in which agents report truthful information. Hence, the best outcome can be reached despite the self-interest of agents. This strategy-proof property is thus a prerequisite for overall efficiency.

iii) **Individual rationality**. To ensure voluntary participation, the profit must be non-negative. Individual rationality is already defined through the definition of imputations in 3.3.4.

iv) **Budget balance**. A budget-balanced mechanism requires that the total payment is greater than zero. This ensures that the market operator never incurs a financial deficit. Sometimes, budget-balance is more restrictive and implies that the market operator has neither a deficit nor a profit. It is known as strong budget-balance.

Remark 3.2. There is a common misinterpretation of efficiency. Traditional market-clearing in electricity markets is achieved through an optimization problem (maximizing the total surplus, cf. 3.8). Therefore, the allocation (based on the provided preferences) is efficient because the (Nash) equilibrium corresponds to the optimization solution. However, all players try to define their bids and offers strategically. They are in a competitive market. There is no truthful strategy (incentive compatibility), i.e., players do not necessarily report truthful information. Consequently, the overall market efficiency is not guaranteed (the other market imperfections are discussed in the previous chapter).

It is obvious that the conjunction of these four properties is ideal, but it is theoretically impossible [100]. Indeed, the Hurwicz impossibility theorem states the following:

Theorem 3.5. *(Hurwicz impossibility theorem). It is impossible to implement an efficient, budget-balanced, and strategy-proof mechanism in a simple exchange economy with quasi-linear preferences (utilities).*

Note that quasi-linear preferences enable the transfer of utility across agents via side payments.

3.4.2. Organized market designs

As previously developed in 2.3, uniform pricing and locational marginal pricing are the most widespread designs of economic dispatch. There are, however, clearing mechanisms that raise increasing attention from electricity market actors. Although they are subject to significant barriers to their application, there is currently extensive research and development undertaken in this field. All these designs are also competing in the implementation of local markets.

i) **Uniform pricing and LMP**. It is a type of auction in which identical items or a quantity of homogeneous product is sold at the same price. In the case of electricity, potential buyers submit the energy they need and its price per unit in a sealed bid. Potential producers do the same for the energy they sell. Such an auction style with more than one seller is often referred to as double auction. LMP clears differentiated prices based on the location of the bidding zone because of power constraints.

ii) **Vickrey-Clarke-Groves (VCG) auction**. In a VCG auction, individuals are incentivized to reveal their true values of utility or cost. In a way similar to uniform pricing, buyers and producers submit bids and offers without knowledge of other's valuations. The winner of the VCG auction is the bidder who is willing to pay the highest. However, it pays the amount that the bidder displaced by entering the auction would have paid. If only one item is sold, then it is equivalent to a second-price auction.

	IC	Eff.	IR	BB
Uniform pricing			Yes	Yes
VCG auction	Yes	Yes	Yes	
P2P market			Yes	Yes

Table 3.2.: Summary of desirable characteristics for organized market designs. IC is incentive compatibility, Eff. is allocative efficiency, IR is individual rationality, and BB is budget balance.

iii) **Peer-to-peer market**. Unlike the pool-based markets, peer-to-peer markets can engage any pair of buyer and seller in bilateral trade (cf., [67]). There are abundant types of procedures for such a market structure, but a rational approach consists of organizing this type of market. There are such examples that optimize the social welfare as in regular spot markets, but with differentiated prices for each trade and a decentralized computation of the equilibrium.

In the light of the desirable properties, Table 3.2 characterizes the aforementioned market designs. Hence, uniform pricing (or LMP) is budget-balanced and fulfills individual rationality, but the overall market efficiency is not guaranteed. Indeed, as stated in Remark 3, although it features allocative efficiency, the lack of incentive compatibility leads to strategic plays resulting in overall inefficiency. This is why such markets are often modeled as a Stackelberg (leader-follower) equilibrium instead of a pure economic (Walrasian) equilibrium. Peer-to-peer organized markets have the same characteristics because they are equivalent problems with a different formulation (they apply differentiated prices, however). In contrast, VCG is an incentive-compatible mechanism that meets overall efficiency and individual rationality but does not guarantee budget-balance. This last consideration prevents its large-scale implementation because market operators could incur losses.

3.4.3. Cost-based billing designs

The cost-based equilibria do not arise from an organized market mechanism. Instead, they derive from billing scenarios for which demand is considered inelastic.

Using the game-theoretic tools (non-cooperative, cooperative, and mechanism design), it is possible to design cost-based exchange scenarios and corresponding billings. We distinguish:

- **Strategy-proof scenarios**. In such scenarios, every agent has the same cost-minimizing strategy for all strategies of other agents. Additionally, agents are imposed to form the grand coalition. Incentive compatibility is thus enforced for these scenarios.

	IC	Eff.	IR	BB
Strategy-proof	Yes	Yes	Yes	Yes
Nash incentive			Yes	Yes

Table 3.3.: Summary of desirable characteristics for cost-based billing designs. IC is incentive compatibility, Eff. is allocative efficiency, IR is individual rationality, and BB is budget balance.

- **Nash incentive scenarios**. These scenarios correspond to agents adopting strategies leading to the Nash equilibrium. They are not incentive compatible.

In cost-based billings, budget balance and individual rationality are systematically ensured. The first is due to the nature of the billings, which pass on the payments directly from one actor to the other. If the demand is considered inelastic, then the utility is infinite, and individual rationality is guaranteed. Moreover, by definition, strategy-proof designs are incentive compatible.

Although strategy-proof designs feature more desirable characteristics than the Nash incentive a priori, the latter could incentivize higher flexibility levels and overall better solutions. In this case, however, the perfect rationality of the agents is dropped.

This work adopts cost-based scenarios with both strategy-proof and Nash incentive designs. Indeed, as stated in objective 2 (cf. 1.2), one leading thread is recognizing and accounting for the best image of the incurred "truth" cost. Responsible energy communities are defined to minimize that cost instead of relying on market-based prices, which usually are uncorrelated with any tangible truth, other than the one reflected by the intersection of an offer-demand curve.

3.5. Conclusions

This chapter has introduced the key theoretical concepts that underlie the scenarios developed in the remainder of this work, in particular in the scope of responsible energy communities. Generally speaking, the relevance of energy communities is conditioned upon reaching an efficient equilibrium. In our responsible proposal, the common denominator of each equilibrium is economic, but the costs at stake already include other societal challenges, as previously introduced in chapter 2 (e.g., environmental). Therefore, mathematical and economic notions should objectivize efficiency and equilibria.

Firstly, the fundamentals of convex optimization were presented. Indeed, the retained assumptions and relaxations of the scenarios introduced in the following chapters, allow us to use this well-developed theory without loss of generality. Then, an insight into game theory was provided to characterize and manage the strategic interactions existing in the energy collectives defined further. Finally, organized market designs and cost-based billing designs are presented and compared, in light of the desirable economic properties. Hence, the pros and cons of each contribution can be better grasped.

Armed with these notions, the next chapters focus on developing various scenarios of energy exchange scheduling, which result ultimately in establishing responsible energy communities.

Box 3.2: Connecting variational inequality and complementarity theories

The finite-dimensional non-linear complementarity problem consists of a system of a finite number of inequalities subject to the requirement that its variables are non-negative and orthogonal to these inequalities [92]. It lies at the foundation of constrained optimization problems.

Variational inequalities and complementarity theory are strongly connected, the former being a generalization of the latter [93]. Both theories were, however, developed independently in their early stage and aimed at meeting different needs. Originally, variational inequalities addressed equilibrium problems that involved partial differential systems, such as in elasticity and plasticity theories as well as mechanics (cf. Signorini problem [94]). Later, finite-dimensional variational inequalities and non-linear complementarity problems (NCP) were studied in the scope of equilibrium programming. Equilibrium programming is a subfield of mathematical programming that computes economic equilibria. This subfield relied until then on fixed-point methods, which were strongly limited for solving real problems. Later on, the connections between both theories allowed significant advances and the development of efficient algorithms.

In its most general form, the complementarity problem is equivalent to a variational inequality when the set Ω corresponds to a cone and $\mathbf{F}(x)$ belongs to the dual of cone Ω [91]. Complementarity problems encompass a wide range of particular forms that have an interest in various fields. Among the main types, we find:

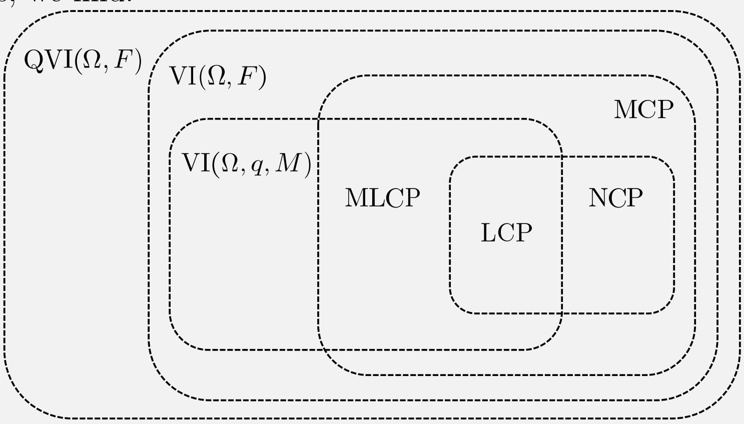


Figure 3.7.: Connections between CPs and VIs.

i) **Non-linear complementarity problems (NCP).** They are a complementarity problem whose cone corresponds to the non-negative orthant of $\mathbb{R}^n$.

ii) **Mixed complementarity problems (MCP).** The system of inequalities and the complementarity condition of an NCP are supplemented by a set of equality constraints. MCPs frequently arise in convex optimization problems through their KKTs (e.g., market clearing).

iii) **Linear complementarity problems (LCP).** LCPs are a special case of NCP in which $\mathbf{F}(x)$ is linear. They were made popular because the KKTs of a linear or a quadratic program coincide with such a formalism. Popular algorithms such as the interior-point method are based on the LCP formulation. Besides, the derivation of an LCP formulation for bimatrix games raised much interest [95].

iv) **Mixed linear complementarity problem (MLCP).** They correspond to an LCP supplemented by a set of equality constraints.

Remark 3.1. $\mathrm{VI}(\Omega, q, M)$ refers to variational inequalities whose function $\mathbf{F}$ is an affine function given by $\mathbf{F}(x) \equiv q + Mx$.

Part II.

Truth Cost, Aggregation and Fairness

CHAPTER 4.

Energy consumption scheduling in liberalized electricity markets

In the first three chapters, the basic concepts and contextual aspects of responsible energy communities were introduced. Their relevance in the electricity supply landscape was justified, and the key theoretical concepts they involve were defined.

This chapter unveils the core proposition of the thesis and will serve as a basis for achieving responsible energy communities. It consists of a unique collaborative scenario of day-ahead energy consumption scheduling.

The first key feature is to provide the right signals to coordinate flexible participants optimally. Only one network infrastructure feeds all the members of a community. Because the overall state of the grid consists of the superposition of all end-user behaviors, distribution and transmission costs should be aggregated at the community level. The proposed responsible energy community model goes in this direction and adopts concerted energy consumption scheduling scenarios (cf. 2.5.3).

Furthermore, the advanced liberalization process that has taken place in Europe cannot be overlooked. In most European countries, it has offered the opportunity to individuals to source their energy from the supplier of their choice on a competitive market. Hence, the core proposition accounts for this complex reality and respects the philosophy of the liberalized structure as found in Europe.

This chapter hence develops scenarios incorporating these two key features. They are at the foundation of the responsible energy community model, which includes additional original features. These developments are presented in the next chapters.

Note that this chapter's content corresponds to the main results published in the *IET Generation, Transmission & Distribution* journal

article "A Cooperative Demand-Side Management Scenario for the Low Voltage Network in Liberalised Electricity Markets" [24].

In the remainder of this chapter, the current state of the literature is exposed, and the motivations are recalled in Section 4.1. Section 4.2 introduces the system structure and the base model that underlies all the collaborative scenarios presented in this work. Section 4.3 details the scenarios, their formulation, and how billing schemes can be derived. It also characterizes the game that governs the interactions between end-users and presents the decentralized algorithm. Section 4.4 compares the outcomes of the different cost distributions, i.e., billing, among community members through a set of benchmarks. Finally, section 4.5 discusses the scenarios and results.

NB: Indiscriminately to all collaborative scenarios, the collectives comprising the participating end-users are referred to as communities. Responsible energy communities refer to the specific scenario developed in chapter 6.

4.1. Introduction

As discussed in the previous chapters, unlocking the flexibility distributed throughout the different levels of the electricity system can be a decisive step for a successful energy transition. The development of demand-side and load management programs, the integration of distributed energy resources, and the deployment of smart grid technologies give new possibilities for the system, provided that the right incentives are adopted.

As exposed in 2.4.3, energy consumption scheduling consists of programming time flexible devices, e.g., dishwasher, electric vehicle, washing machine to answer an objective function, which usually features a dynamic billing. There is a vast literature on the subject. Some cooperative schemes allow aggregating scheduling flexibility, hence giving more optimization potential. However, a challenging aspect of such scenarios is to propose a fair and incentivizing cost distribution.

Various contributions address these considerations in scenarios involving groups of interacting members [101]–[111]. Among them, game theoretical approaches are commonly used when strategic behaviors are possible.

On the one hand, non-cooperative (strategic) games are formulated when interactions are not subject to binding agreements (cf. 3.3.1), such as in [101]–[103], [106]–[109]. By designing the game appropriately, it is possible to make cooperation self-enforcing through its Nash equilibrium. In [101], an aggregative game solving the day-ahead energy consumption scheduling for a set of users owning time flexible appliances is formulated. The problem is solved using an autonomous and decentralized best-response algorithm, and it is shown that the outcome is equivalent to that of the global optimization, i.e., the social optimum. These formulations, however, lack the possibility of incentivizing participants to consent to higher flexibility. The introduction of distributed energy resources combined with the use of an hourly distribution of the costs in [107] addresses this issue. The Nash equilibrium is computed using a proximal decomposition algorithm, but higher flexibility is obtained at the cost of a non-socially optimal solution [106].

On the other hand, cooperative game theory focuses on incentivizing the formation of coalitions (cf. 3.3.4), such as in [110], [111]. In such schemes, cooperation is based on a predefined consensus enforced by a supervising authority. In [110], end-users form coalitions for trading a portion of their electricity. Their participation is optimized by relying on the Shapley value for adequately distributing energy savings. Han et al. [111] propose a similar approach, which is complemented with an energy management optimization. Such a predefined consensus allows for explicitly defining how costs are distributed among end-users, thus ensuring fairness within the community. However, several shortcomings still had to be addressed in the literature.

Firstly, the related works focus on commodity costs, and neglect network-related fees, while the latter contribution represents considerable expenditures within the electricity bill 2.3. Secondly, they overlook the current liberalized structure of European electricity markets.

In this chapter, the collaborative mechanism as in [101] is taken further and adapted to the more complex reality of liberalized electricity market. **The objective is to efficiently schedule the energy consumption among consumers of the same cluster of feeders, i.e., behind the same MV/LV transformer, on a day-ahead basis in a European liberalized setting**. End-users can freely choose their energy supplier in a competitive market, but they are subject at the same time to transmission and distribution network fees associated with a fixed transmission system operator and distribution system operator following their geographical location. The two main costs associated with electricity supply are thus the energy generation (commodity) costs and network use costs. Real-time pricing is considered for the former, while power-based pricing echoes the costs of the latter.

Whereas it is easy to transpose the autonomous DSM program of [101] to the commodity cost among the customers of a given electricity supplier, the literature had not tackled the optimization of both the network costs and the commodity costs under several suppliers. Indeed, the network costs are attributable to all users without any distinction of electricity supplier, and the optimized solution usually does not match those of the energy suppliers. This highlights the need for an additional collaborative mechanism between all users of one network that is put forward in this chapter.

The main contributions presented in this chapter are summarized as follows.

i) A collaborative mechanism between end-users delivered by different electricity suppliers is presented. The mechanism minimizes the sum of the costs associated with the common network use and the energy generation by each supplier (i.e., the social optimum).

ii) The optimization problem is formulated, and it is shown how it can be conveniently solved by all the participating consumers using a decentralized algorithm, thus ensuring more privacy.

iii) A method for distributing the total cost fairly between the pools of the different electricity suppliers is proposed. It is supplemented with further distribution among their end-users. The billing ensures that the underlying "Energy Consumption Scheduling" (or ECS) game between users has a unique Nash equilibrium.

iv) Different levels of cooperation are compared, from completely passive users to entirely cooperative users, in terms of costs and load profile.

v) The most and least favorable circumstances for cooperation are studied and discussed.

4.2. Common framework and elementary model

This section outlines the shared structure **for all the collaborative scenarios developed in this work.** The main guidelines, hypotheses, and possible interactions between end-users are introduced. Furthermore, the elementary models for the energy consumption scheduling are presented. All these elements are further particularized in the next chapters as it involves additional considerations.

4.2.1. Guidelines, hypotheses and frameworks

In light of the many considerations mentioned in the previous chapters, and in addition to them, a set of guidelines and hypotheses have been defined. Together, they form the core substance of this work.

The main guidelines that the scenarios hold are the following.

i) The **optimization of the "truth" total cost** of delivering electricity to a defined collective of prosumers by coordinating power flows among them.

ii) A scenario design **enhancing participation and commitment** inside a coherent collective of consumers and prosumers.

iii) A **fair cost distribution** accounting for participation efforts and respectful of economic disparities.

iv) The **impossibility** by community members **to make a profit**, hence complying with the first guideline. Note that all scenarios aim cost reductions.

v) The **promotion of more sustainable energy sources** and solutions as well as more local consumption.

Depending on the degree of enforcement by each scenario, the collectives of prosumers may be termed as responsible energy communities (cf. Chapter 6). In addition, we adopt within this framework the following main hypotheses:

i) A context of liberalized electricity markets with its related actors as we know today in Europe (suppliers, system operators, prosumers...), cf. 2.2.

ii) A day-ahead demand-side management scenario for a community, located on a low-voltage network, based on energy consumption (or exchange) scheduling.

iii) A real-time pricing structure for commodity costs and power-based pricing for network costs (cf. 2.4.2).

iv) A decentralized resolution of the scheduling suited to smart meters and smart grid communication possibilities while maintaining privacy.

v) An aggregated network cost among all community actors for all scenarios is considered.

vi) The mutualization and sharing of the excess locally produced energy and the excess storage capacity inside the community are assumed in the energy exchange scenarios developed in Chapter 6.

vii) A non-elastic demand, i.e., the total energy needs at the end of the day are constant (even for the flexible load).

viii) All end-users connected to the community network must participate and abide by the rules of the community.

These hypotheses are detailed and particularized in all the scenarios further developed in this work.

Table 4.1 summarizes the frameworks assumed in each chapter. They are distinguished according to the mathematical methods, i.e., convex optimization or variational inequality theory, and the extent of the scenario. The latter is either the basic scenario where only coordination is involved (corresponding to "Energy Consumption Scheduling") and the mutualization scenario where excess resources are shared among participants (corresponding to "Energy Exchange Scheduling", or EES), cf. 2.4.3 and Figure 2.7.

Table 4.1.: Summary of chapters' framework coverage in function of mathematical methods and the extent of the scenarios.

Involved chapters	Convex optimization	Variational inequalities
Basic scenario (ECS)	4 5 (storage included)	5
Mutualization (EES)	6	(6) perspective

To conclude this paragraph, it is worth mentioning two particular attention points:

- *Uncertainty.* All methods developed in this work assume a deterministic framework. In practice, uncertainties are inevitable. For instance, the operation of a device may be postponed, canceled, or initiated in advance. It does not disqualify the essence of the methods and the main observations, but realistic conditions will lead to reduced expected performances. One convenient solution that can limit the loss of efficiency is to opt for robust approaches (cf. prospects in 8.2.

- *Time window.* The energy consumption scheduling is always achieved on a one-day (24 hours) horizon. This has obvious advantages, such as concurring with the day-ahead market or providing an obvious tradeoff between a sufficiently long enough window to make significant optimization and a limited horizon to prevent increasingly likely scheduling changes. However, there are significant limitations. For example, planning devices with operation spanning over two different days is not efficient. It is possible to mitigate that effect by provisioning part of the energy needs for one day and the other part for the next based on past statistics.

4.2.2. Actors and their roles

As liberalized electricity markets are assumed, we recall and specify the functions of the actors directly involved in each scenario.

End-users

They are the consumers or prosumers forming the community. They are connected to the nodes of the corresponding low-voltage distribution network and are thus mainly residential or small and medium business users. They can feature very heterogeneous consumption profiles as they have, for example, different hours of occupation, various devices, and habits. In line with the liberalized context, customers retain free choice over their energy supplier, and they are inclined to consent to more or less consumption flexibility according to the nature of their activity, preferences, and community sensibility.

End-users comprise up to four different electric components.

- *Non-flexible devices.* This is the portion of the consumption for which users do not wish to consent to any flexibility (e.g., lighting, fridge, TV, computer).
- *Non-flexible generation.* They are mainly the photovoltaic panels and represent a negative consumption without any flexibility.
- *Flexible devices.* These are devices for which time constraints are usually applicable but allow more or less flexibility for the load scheduling (e.g., dishwasher, electric vehicle, washing machine).
- *Energy storage systems.* Although electrochemical batteries (Lithium-ion technology) are on the rise for home energy storage solutions, there are many other serious prospects such as thermal storage or power to gas.
- *Flexible generation.* They are small controllable power generators such as micro combined heat and power or fuel cells. Although flexible generation is not included in this work, it would not change the methodology. Indeed, it can be seen as a negative consumption with time flexibility.

In this work, end-users seek to minimize their electricity bill. Although they have control over the commodity costs, the network tariff scheme impedes any individual optimization because the actions of all the end-users contribute to define the network costs, cf. 4.2.4. Collaborative scenarios are, therefore, relevant to that aim.

System operators

Transmission and distribution system operators operate over a circumscribed area. They usually own all the electric lines and cables in the same region. They have a de facto monopoly and are consequently strongly regulated by states. End-users are thus bound to their geographical TSO and DSO. Although the network tariffs they apply are usually proportional to the total energy consumed and sometimes also to the peak power (cf. CPP in 2.4.2), it appears that they do not reflect the real cost of today's reality. Whereas in the past, system operators sought only to maintain and renew their infrastructure on a time basis with little or no effect of the consumption variations, the more stringent operating conditions of the network introduced by decentralized generation and the expected consumer-centric approaches lead to operational issues such as voltage range infringements or congestions.

Active management of the network depending mostly on power flows will be a major focus point in the short term. The cost for system operators is optimized when load flows are minimized and most uniform throughout the day. Hand in hand with regulators, they should therefore develop a new tariff structure reflecting that reality. This consideration is included in the cost structure adopted for the scenarios as detailed further in 4.2.3.

Electricity suppliers

In the context of liberalized electricity markets, numerous suppliers can offer electricity to the users. Suppliers usually have different selling strategies depending on the generation portfolio they hold and their market positioning. To consume energy more efficiently, dynamic pricing could become widespread. The popular day and night rates are a simplistic application of time-of-use pricing. However, due to the high variability of renewables, prices should be much more granular (cf. 4.2.3).

Suppliers should offer transparent tariffs carefully reflecting the energy mix in their portfolio, its dynamics, and the variations due to forecasts and market prices. It is an important consideration to increase the penetration of renewables healthily. Consequently, customers can choose the supplier with the right pricing time grid suiting their needs at the lowest cost, and thus helping at consuming energy more rationally. The collaborative scenarios presented in this work are developed following that aim.

The community

More than a collection of individuals cooperating on the same low-voltage network, the community is regarded as a single entity on the electric system. Consequently, the exchanges inside the community are invisible towards the rest of the system. Only the net offtakes at the common injection point (e.g. MV/LV transformer) are billed and passed on to community members, as described further in each scenario. Moreover, some scenarios assume that the excess local production and storage are mutualized and made available to the rest of the community (cf. Chapter 6). In that manner, it ensures the attractiveness of such investments. Indeed, the benefits are twofold:

i) free energy is made available to those in need,

ii) the community net imports are reduced.

4.2.3. An ECS scenario under real time pricing

Principle

The following paragraphs recall the primary notions that are featured for the scenarios developed throughout this work.

If we assume that the pricing applied by the actors should reflect the actual cost of electricity all along the supply chain, then cost represents the most obvious variable to optimize. Still, the true cost of electric energy is not constant with time. If it is to be used more rationally, consumers should be aware of it. This is how different types of smart pricing exist and can be employed as signals for the DSM programs, cf. 2.4.2.

Among the leading smart pricing schemes, time-of-use pricing allows capturing daily patterns of generation and demand, hence suiting particularly the dynamics of a traditional generation mix characterized by its higher controllability. In a context of higher decentralized generation, the dynamics changes day after day, and tariffing schemes such as critical-peak pricing or real-time pricing are better suited. The first aims at preventing heavy-load conditions during critical periods of the day, whereas the second reflects as closely as possible the current generation costs. Hence, RTP defines prices on a short time interval basis (e.g., an hour or a quarter of an hour) usually communicated one day in advance. These different types of dynamic pricing are enhanced by the ongoing large-scale deployment of smart metering units and the emergence of the smart grid concept in all of Europe.

Energy Consumption Scheduling is a demand-side management technique in which flexible loads (e.g., dishwasher, electric vehicle, washing machine) are programmed to answer an objective function. In this work, the objective is to minimize both commodity and network costs based on the RTP tariff. Hence, collaboration is needed across the entire community as they share the same network. In the more elaborate scenarios of Chapter 6 leading to responsible energy communities, energy transactions take place between the participants. The counterpart of ECS in such a setting is referred to as Energy Exchange Scheduling (EES).

Dynamic pricing and energy consumption scheduling are detailed respectively in 2.4.2 and 2.4.3.

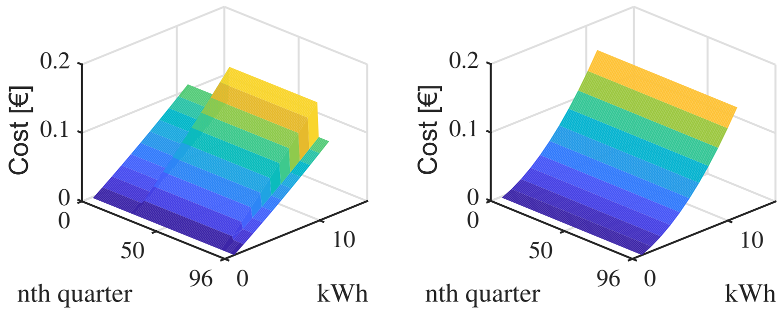


Figure 4.1.: Example of a supplier cost curve (left) and a DSO cost curve (right) with a 15 minutes time resolution.

Cost curves

For all the collaborative scenarios, it is assumed that the cost curves (in €) are defined by each actor on a day-ahead basis. They comprise one cost function for each interval of the day considered (e.g., 15-min time intervals is a commonly adopted interval resolution by smart meters). Instead of having one constant price (in € /kWh) leading to a passive attitude of end-users, applying a higher granularity and a possible progressiveness based on their consumed energy can help shape the desired behavior and implement DSM scenarios. However, the curve shape must reflect the reality of the underlying costs.

A pricing scheme differentiated in the level of consumption (cf. Figure 4.1, right) is considered for the network contribution. It is applied to the aggregated load. Indeed, system operators do not have any degree of flexibility, and they should reflect the utilization burden as it is. The consequences of power flowing in the network remain the same throughout time; however, they are enhanced depending on the solicitation level. Indeed, charges such as power losses or the wear and tear of assets increase superlinearly. For example, cable ohmic losses increase with the square of the current. Hence, we define a cost curve constant in time and increasing quadratically with the consumption, i.e., prices (in €/kWh) increase linearly. This last assumption is a sweeping simplification of the true network cost. Yet, it unleashes collaboration potential because prices are load-dependent and end-users gain from coordinating their consumption.

On the other hand, energy suppliers have access to some flexibility. Depending on their portfolio content (e.g., generation assets, market participations), the energy mix is characterized by different time-dependent costs. However, it is chosen not to differentiate the prices based on their level of consumption. Hence, the applied costs to individuals are linear (i.e., prices are constant), as suppliers have some degree of freedom and effective tools at their disposal, and also for the sake of simplicity towards consumers (cf. Figure 4.1, left). Indeed, they should be able to select easily the plan that best suits their needs. The higher complexity that those curves can hold should be carefully governed by a few simple indicators that make it possible to compare easily different contract offers. Among those, the most straightforward indicators could be:

- the *mean price*,
- the *minimum and maximum prices*,
- *variability indicator(s)* etc.

Alternatively, suppliers could be simple intermediaries for end-users to access the spot market, which on the day-ahead horizon behaves similarly to RTP.

The details on pricing are provided in 4.2.4 and particularized to the scenarios developed in the next chapters.

4.2.4. Elementary model

Note that in the following, curly inequality operators (e.g., $\succeq$) are used for component-wise inequalities and the delta-equal-to symbols ($\triangleq$) define variables or sets.

Power network model

In order to adopt a scenario in which network considerations are fully integrated, it is assumed that community members are connected to the same low-voltage entity (such as a distribution feeder or a collection of them).

The proposed design can be seamlessly integrated into the current liberalized framework: the community network is managed by a single DSO, while each member n can choose among different suppliers for its net consumption $\mathbf{l}_n$ (accounting for the mutualization of resources with other end-users).

In the remainder, the following sets are addressed.

- The set of *community members*, denoted with $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$.
- The set of *members who contracted the same supplier* i, denoted with $\mathcal{N}_i \triangleq \{1, 2, \ldots, N_i\}$.
- The set of *optimization intervals* for one day, denoted with $\mathcal{T} \triangleq \{1, \ldots, T\}$, each of duration $\Delta\tau$.

Prosumer load model

For each community member $n \in \mathcal{N}$, different load components can be distinguished (cf. 4.2.2). Depending on the scenarios, the following variables are adopted in the mathematical formulations.

- The *flexible consumption*, i.e., the portion of the load for which prosumers consent flexibility in their operation (e.g., washing machine, heat pump, electric vehicle). This set of appliances, denoted by $\mathcal{A}_n$, can be operated at the most opportune time under intertemporal constraints (some processes have fixed load profiles and/or cannot be stopped during operation). Each appliance $a \in \mathcal{A}_n$ is described by a power scheduling vector:

$$\mathbf{x}_{n,a} \triangleq \left[x_{n,a}^1, \dots, x_{n,a}^T\right] \succeq 0, \tag{4.1}$$

  which is formed by the collection of the corresponding values at time intervals $t \in \mathcal{T}$.

- The *battery storage* whose charge and discharge schedule is described by the storage vector:

$$\mathbf{s}_n \triangleq \left[s_n^1, \dots, s_n^T\right]. \tag{4.2}$$

- The *non-flexible load* includes critical appliances (e.g., lighting, kitchen equipment, multimedia devices). Its forecasted power vector is defined by:

$$\mathbf{k}_n \triangleq \left[k_n^1, \dots, k_n^T\right] \succeq 0. \tag{4.3}$$

- The *non-dispatchable generation*, i.e., locally produced energy by photovoltaic (PV) panels and small wind turbines. The predicted generation vector is denoted by:

$$\mathbf{g}_n \triangleq \left[g_n^1, \dots, g_n^T\right] \succeq 0. \tag{4.4}$$

The non-flexible load and the photovoltaic generation cannot be controlled. In the context of a deterministic approach, they can thus be treated as state variables for the day-ahead procedure. They can thus be grouped as follows.

- The *baseload vector*. It is obtained by simply substracting the generation from the non-flexible load:

$$\mathbf{d}_n \triangleq \mathbf{k}_n - \mathbf{g}_n. \tag{4.5}$$

Furthermore, we use Λ_n to denote the set of decision variables of prosumer n, including $\bigcup_{a \in \mathcal{A}_n} \mathbf{x}_{n,a}$ and $\mathbf{s}_n$, and Λ^t to denote the set of decision variables at time

t. Finally, the union of all the decision variables across all the prosumers and time slots is defined as $\Lambda \triangleq \bigcup_{n\in\mathcal{N}} \Lambda_n = \bigcup_{t\in\mathcal{T}} \Lambda^t$.

The formulations of the scenarios include various load aggregation levels. We refer accordingly to

- The *individual net load*, metered by the DSO at time step $t \in \mathcal{T}$ (Figure 5.1) is defined as:

$$l_n^t = d_n^t + x_n^t + s_n^t. \tag{4.6}$$

- The *total load across all users of the same supplier* i cooperative pool at time step $t \in \mathcal{T}$ is:

$$\sum_{n\in\mathcal{N}_i} l_n^t = L_i^t. \tag{4.7}$$

- The *total community load* at time step $t \in \mathcal{T}$ is denoted by:

$$\sum_{n\in\mathcal{N}} l_n^t = L^t. \tag{4.8}$$

Additionally, the load scheduling variables are subject to individual requirements and physical constraints. We thereby characterize the following.

- The *scheduling flexibility* for each appliance a of member n by a permissibility vector:

$$\boldsymbol{\delta}_{n,a} \triangleq \left[\delta_{n,a}^1, \ldots, \delta_{n,a}^T\right], \qquad \delta_{n,a}^t \in \{0,1\}. \tag{4.9}$$

  This binary vector specifies the time intervals during which the member agrees for its appliance to be scheduled (1: permitted, 0: not permitted). If the total predetermined energy consumption of appliance a is $E_{n,a}$, the resulting power profile constraints are:

$$\boldsymbol{\delta}_{n,a}\mathbf{x}_{n,a}^{\mathbf{T}}\,\Delta\tau = E_{n,a}, \tag{4.10}$$

$$\mathrm{NOT}(\boldsymbol{\delta}_{n,a})\mathbf{x}_{n,a}^{\mathbf{T}} = 0. \tag{4.11}$$

- The *power consumption constraints*. Some appliances have predetermined consumption cycles (e.g., washing machine) while others have modular cycles (e.g., electric vehicles). For the sake of simplicity and without loss of generality, it is considered that flexible consumption is fully modular. Hence, each appliance is limited only by a maximum power level $M_{n,a}$:

$$0 \preceq \mathbf{x}_{n,a} \preceq M_{n,a}. \tag{4.12}$$

Finally, when storage is considered, a simplified model neglecting all losses is adopted.

- The *battery state of charge constraint*:

$$0 \leq \sum_{t=1}^{\bar{t}} s_n^t \Delta\tau + E_{\mathrm{st},0} \leq E_{\mathrm{st}}, \qquad \forall \bar{t} \in \mathcal{T} \tag{4.13}$$

  where $E_{\mathrm{st},0}$ is the initial state of charge and E_{st} is the storage capacity.

- The *maximum charge and discharge levels*, respectively denoted by M_n^{ch} and M_n^{dis} limit the operation of the battery. They yield:

$$-M_n^{\mathrm{dis}}\mathbf{1} \leq \mathbf{s}_n \leq M_n^{\mathrm{ch}}\mathbf{1}, \tag{4.14}$$

  where $\mathbf{1} \triangleq [1, \ldots, 1]$ is the all-one vector of length T.

Electricity cost and pricing model

The day-ahead energy consumption/exchange scheduling problem is based on the minimization of the electricity costs, which consist of two contributions in the objective function.

- The *commodity costs.* They are associated with the marginal generation cost and are billed by the suppliers. Each member of the community can freely choose its supplier $i \in \mathcal{S} \triangleq \{1, \ldots, n_s\}$. However, based on the consumption history and the prosumer profile, it is easy to recommend or automatically attribute the best-suited supplier. It is assumed that energy is billed by suppliers according to a dynamic pricing scheme. Prices are not considered to be power-based, i.e., they are independent of the load level, as the suppliers have a certain degree of control on their costs via their generation portfolio and market positions. This also has the advantage of providing a more legible comparison between suppliers. Besides, only the positive part of the individual net load $l_n^{t+} = \max(l_n^t, 0)$ is billed (the negative part l_n^{t-}, which is generated at zero marginal cost, is valorized among the community members). Therefore, supplier i applies for each of its customer $n \in \mathcal{N}_i$:

$$C_{\mathrm{supp},i}^t(l_n^t) = \gamma_{\mathrm{com},i}^t l_n^{t+} \Delta\tau, \tag{4.15}$$

where $\gamma^t_{\text{com},i}$ is the commodity price (in €/kWh) of supplier i at time t. Within the community models we propose, the commodity costs associated with each individual can be influenced by their contribution regarding flexibility, which is defined by the joint scheduling. Also, there can be internal transactions between members (cf. Chapter 6). To fairly account for these considerations, it can be necessary that the final contribution of client n differs from $C^t_{\text{supp},i}(l^t_n)$. In that respect, different ways to distribute the total costs $\sum_{n\in\mathcal{N}} C^t_{\text{supp},i}(l^t_n)$ at the community level are presented in Section 4.3 and in the next chapters. It should be noted that this costs distribution (within the community) does not affect the final revenues of the supplier i, i.e., $\sum_{n\in\mathcal{N}_i} C^t_{\text{supp},i}(l^t_n)$.

- The *grid costs.* They are associated with the power flows at the community interface, thus reflecting the effects on transmission and upstream distribution grids. These costs $C^t_{\text{up}}\left(L^t\right)$ are not proportional to the load. Hence, we consider a non-linear cost function depending on the power at the interface node L^t. Without loss of generality, we consider a quadratic cost function:

$$C^t_{\text{grid}}(L^t) = \gamma_{\text{grid}}\left(L^t \Delta\tau\right)^2, \tag{4.16}$$

where γ_{grid} represents the grid fees (in €/kWh^2). Note that in Chapter 6, grid costs have two components. The first are the upstream grid costs, which are similar to the above, and the second are the community grid costs, which are related to the internal flows of the microgrid formed by the community.

4.3. Methodology

In this section, the methodology of the core collaborative scenario referred to as *inter-supplier ECS* is exposed. It should be recalled that it includes two key features. **It introduces a billing structure that reflects as much as possible the truth incurred costs and the liberalized electricity supply framework, respectively.** Other more advanced scenarios are presented in the following chapters, as summarized in Table 4.1.

Firstly, the overall philosophy of a few basic energy scheduling scenarios is presented along with the core proposition. Then, the possible interactions between community members and the required communication means are described.

4.3.1. Benchmarks

In order to decrease energy consumption costs, the processing capability introduced by smart meters or third party equipment is used to set up a collaborative scheduling mechanism between end-users of different suppliers on the same low-voltage network. To assess the proposition, it is convenient to benchmark some results with different intermediate situations. Hence, we choose to review various scenarios that include progressive cooperation and proactivity on the end-user side. The scenarios considered are the following.

i) **Passive demand.** The end-users take absolutely no reactive actions towards the price signal. Despite the varying prices, they act as if they were subject to fixed fares. They consume energy starting at the beginning of the respective permissible time intervals of each device, cf. (4.9).

ii) **Flattened demand.** The end-users modulate their appliances to consume at constant power during the whole respective permissible time intervals attributed, thus guaranteeing a low individual peak-to-average ratio.

iii) **Intra-supplier ECS.** The end-users of a given collaborative supplier pool schedule their load together to minimize their joint commodity cost.

iv) **Inter-supplier ECS.** The users of all collaborative supplier pools, i.e., the community, schedule their load to minimize both the commodity and network costs. This is the main contribution of this chapter.

Passive demand and flattened demand are respectively a pessimistic (individuals do not intent to adapt their consumption profile) and an optimistic (individuals are fully flexible although they don't have access to grid costs) scenario when no ECS is considered. They are both unrealistic, but they can be seen as upper and lower bounds of some realistic consumption schemes. The implementation details about each scenario are provided in the next subsection.

Depending on the scenario considered, interactions and communication between end-users are necessary. All community participants exchange their individual aggregated load through their smart meter (cf. Figure 4.2). That information is necessary only for the ECS algorithm, and it is therefore not stored. In addition, each supplier sends the price information to each of its customers. In return, the aggregated load of each supplier pool is communicated to their respective supplier for billing. All the individual net loads (l_n^t) of the considered network are made available to the DSO, and the network cost curve is applied to the aggregated load L^t. In this manner, no individual load is stored, hence maintaining privacy. The following section formalizes each of these interactions mathematically.

4.3.2. Energy consumption scenarios

The energy consumption scheduling strategies presented in this chapter assume a prosumer load model 4.2.4 that includes flexible consumption, non-flexible load, and non-dispatchable generation. The net load of an individual (4.6) is thus simplified

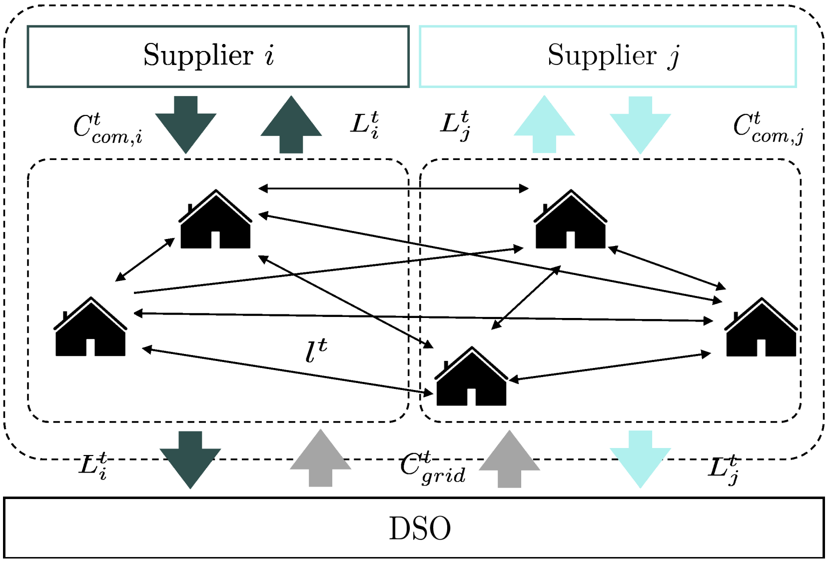


Figure 4.2.: Communication diagram inside the community.

to $\mathbf{l}_n = \mathbf{d}_n + \mathbf{x}_n$. Based on this assumption, the mathematical formulations are provided in the next paragraphs.

Reference strategies

To evaluate the core proposition, the comparison that is provided by the benchmarks presented in Section 4.4 includes two simplistic consumption strategies. Although they are somewhat unrealistic, they give lower and upper bounds for what a non-optimized consumption strategy could yield.

i) **Passive demand.** End-users schedule their load as if each appliance would consume at its maximum power $M_{n,a}$ from the beginning of the respective permissible time intervals until the total energy needed $E_{n,a}$ is consumed. The additional constraint is therefore:

$$x^t_{n,a} = M_{n,a} \quad \forall t : \sum_{k=1}^{t} \delta^t_{n,a} \leq \frac{E_{n,a}}{M_{n,a}\Delta\tau}. \tag{4.17}$$

ii) **Flattened demand.** Consumers are aware that a high level of power consumption is detrimental to the network and can be most prejudicial in terms of costs. In an effort to avoid a high peak-to-average ratio without requiring any optimization capability, they can judiciously modulate the consumption of each appliance so that they consume a constant power over the whole permissible time scheduling intervals. This is formalized by the extra constraint:

$$\delta_{n,a}^t x_{n,a}^t = \frac{E_{n,a}}{\sum_{q=1}^{T} \delta_{n,a}^t} \delta_{n,a}^t \quad \forall t \in \mathcal{T}. \tag{4.18}$$

Energy consumption scheduling strategies

Although the scheduling optimization can aim at solving technical considerations such as the minimization of the peak-to-average ratio, we focus on minimizing energy costs. The intra-supplier ECS minimizes costs attributable solely to the energy generation, whereas the inter-supplier ECS minimizes the total energy cost, including distribution costs for all end-users. As previously discussed, we assume that energy cost functions are carefully designed to reflect the true cost. Besides, we refer to $\Lambda_i \triangleq \sum_{n \in \mathcal{N}_i} \Lambda_n$, the set of all decision variables across a supplier collaborative pool, in the following formulations.

- In the *intra-supplier ECS* scheme, each supplier i minimizes the following function:

$$\begin{aligned} \min_{\Lambda_i} & \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}_i} \gamma_{\text{com},i}^t l_n^{t+} \Delta\tau \\ & \text{s.t. constraints} \quad (4.9) - (4.12). \end{aligned} \tag{4.19}$$

  Problem (4.19) is linear and may have more than one optimal solution achieving the same minimum total cost. We consistently choose the solution that leads, for each device, to the flattest power consumption cycle to limit the underlying network costs. The problem can be solved using a linear programming method, such as the simplex algorithm or the interior-point method (cf. 3.2.3).

- In the *inter-supplier ECS* strategy, the optimization problem integrates the network cost component and is expressed by the following expression:

$$\begin{aligned} \min_{\Lambda} & \sum_{t \in \mathcal{T}} \left(\sum_{i \in \mathcal{S}} \gamma_{\text{com},i}^t L_i^{t+} \Delta\tau + \gamma_{\text{grid}} (L^t \Delta\tau)^2 \right) \\ & \text{s.t. constraints} \quad (4.9) - (4.12), \end{aligned} \tag{4.20}$$

  where $L_i^{t+} \triangleq \sum_{n \in \mathcal{N}_i} l_n^{t+}$. The interior-point method technique, among other convex programming methods, can be used to solve this quadratic problem as well. The solution of Problem (4.20) is unique given the strict convexity of the resulting cost function.

Instead of solving the optimization problems in a centralized fashion, it is interesting to take advantage of the processing power of the smart meter to avoid additional centralized computing resources. Indeed, this functionality allows a distributed computation with a minimum exchange of information between the units.

4.3.3. Energy consumption game

Both ECS optimization problems naturally form a game for which a Nash equilibrium exists under a few assumptions. However, in the intra-supplier scenario, only the commodity cost component is involved. Indeed, there is no collaboration between users of different suppliers on the network cost component, and thus the exchange of information is limited to the smart meter of the same supplier pool. The formulation, in this case, is similar to [101], in which only the commodity cost is considered. We focus on the inter-supplier scenario game for which the collaboration mechanism proposed in this chapter is introduced.

Billing scheme

In order to establish the electric bill, we introduce the following definitions:

- S_i. *Minimum possible commodity cost* of supplier i pool (solution of the intra-supplier ECS), i.e.,

$$\min_{\Lambda_i} \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}_i} \gamma_{\text{com},i}^{t} l_n^{t+} \Delta\tau. \tag{4.21}$$

- S_T. *Total minimum possible commodity cost*, i.e.,

$$\min_{\Lambda} \sum_{i \in \mathcal{S}} \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}_i} \gamma_{\text{com},i}^{t} l_n^{t+} \Delta\tau, \tag{4.22}$$

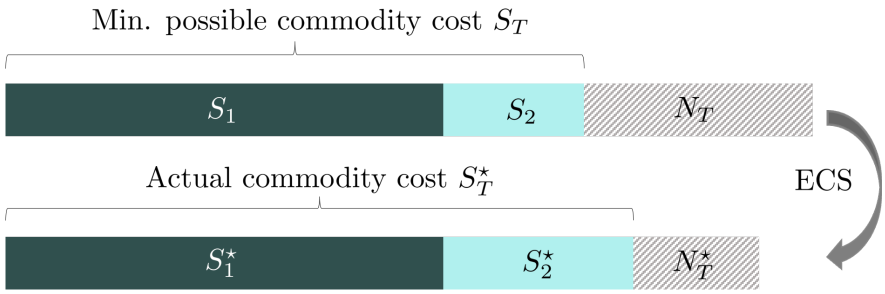
  also corresponding to $\sum_{i \in \mathcal{S}} S_i$.

- N_T. *Network cost deriving from achieving* S_T, i.e.,

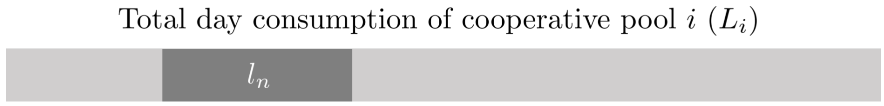
$$\gamma_{\text{grid}} \left(L^t \Delta\tau \right)^2. \tag{4.23}$$

  In contrast to S_i and S_T, N_T does not derive from an optimization problem. The total net load L^t corresponds to the values optimizing (4.21), or equivalently, (4.22).

- $S_i^{\star}$. *Actual commodity cost of supplier pool* i, i.e., derived from optimization problem (4.20).



(a) Commodity and network costs before and after collaboration



(b) Fraction of consumed energy by user n

Figure 4.3.: The costs are distributed according to the initial commodity costs ratios (a) and the fraction of energy consumed inside a given supplier (b).

- $S_T^\star$. *Total actual commodity cost*, i.e., derived from optimization problem (4.20).
- $N_T^\star$. *Network cost associated to* $S_T^\star$, i.e., derived from optimization problem (4.20).
- l_n. *Total day consumption* of prosumer n, i.e.,

$$\sum_{t\in\mathcal{T}} l_n^t. \tag{4.24}$$

- L_i. *Total aggregated day consumption* of collaborative pool i, i.e.,

$$\sum_{t\in\mathcal{T}} L_i^t. \tag{4.25}$$

Issuing the individual bills of inter-supplier end-users is not straightforward. Indeed, reaching the total minimum cost (social optimum) requires the end-users of each supplier pool to deviate from the optimal solutions S_i in terms of commodity cost (cf. Figure 4.3a). Some suppliers deviate more or less from what would be their respective optimum (S_i to $S_i^\star$) to decrease the network costs (N_T to $N_T^\star$) and the subsequent total cost across all cooperative users. This leads to a higher total commodity cost $S_T \leq S_T^\star$. More importantly, it distorts the relative contribution of each supplier to the total cost when $S_i^\star/S_j^\star \neq S_i/S_j$.

Passing on the respective costs of each supplier after optimization as it is (proportionnally to $S_i^\star$ would be inequitable. Hence, to apply fair individual billing, a twofold distribution of the total cost is proposed.

i) The **distribution between suppliers** according to the proportion between what would be their minimum related cost if their users would schedule only in function of the energy cost and the underlying total commodity cost, i.e., the first fraction of (4.26). This corresponds to the optimal solutions of the respective intra-supplier ECS.

ii) The **distribution between the end-users** of each supplier proportionally to their fraction of total daily consumption in their respective supplier pool (cf. Figure 4.3b), i.e., the second fraction of (4.26).

Let's denote by b_n, the daily bill, i.e., both the network costs and commodity costs, charged for each user $n \in \mathcal{N}$ at the end of one day. Resorting to the above definitions, it can be expressed as

$$b_n = \frac{S_i}{S_T}\frac{l_n}{L_i}(S_T^* + N_T^*), \quad \forall n \in \mathcal{N}_i. \tag{4.26}$$

For the sake of simplicity, (4.26) becomes

$$b_n = \Omega_n(S_T^* + N_T^*), \tag{4.27}$$

with

$$\Omega_n \triangleq \frac{S_i}{S_T}\frac{l_n}{L_i}. \tag{4.28}$$

This type of distribution will be later referred as a *net load proportional billing*; a type of *daily proportional billing*, cf.5.3.1.

The billing system introduces a distribution between users naturally leading to a game with a unique Nash equilibrium, as further described in the following paragraph. Furthermore, we see from (4.21)-(4.22), and (4.26) that the system operators and the suppliers only need respectively L^t and their corresponding L_i^t for billing purposes. As previously introduced (cf. 4.3.1), no individual load information is thus communicated apart from a total consumption index, denoted by e_n.

Game model

The ECS scenarios involve interactions between self-interested individuals. Indeed, community members aim at getting the lowest individual bill possible. It is thus

Box 4.1: ECS game summary

- *Players*: each prosumer $n \in \mathcal{N}$ is a player who competes against the others.
- *Strategies*: the strategy profiles Λ_n correspond to the possible energy consumption schedules x_n that each prosumer can choose.
- *Payoffs*: they correspond to the individual bills that prosumers want to minimize (the change in sign is due to the fact that payoffs are always maximized in game theory), i.e.,

$$P_n(\Lambda_n; \Lambda_{-n}) = -b_n = -\Omega_n(S_T^* + N^*) \tag{4.29}$$

where Ω_n is the twofold distribution described by (4.28).

meaningful to formulate the ECS problem as a non-cooperative game (cf. 3.3), independently of the chosen billing method.

Let $\Lambda_{-n} \triangleq \{\Lambda_m\}_{m\in\mathcal{N}\setminus\{n\}} \triangleq [x_1, \dots, x_{n-1}, x_{n+1}, \dots, x_N]$ denote the set of all the strategy profiles except those of player n. The game is formally defined by the tuple $\mathcal{G} \triangleq \langle \Lambda, \mathbf{b} \rangle$, where $\Lambda \triangleq \prod_{n\in\mathcal{N}} \Lambda_n$ is the joint strategy set, with Λ_n being the individual strategy set of prosumer n, and $\mathbf{b} \triangleq \big(b_n(\Lambda_n, \Lambda_{-n})\big)_{n\in\mathcal{N}}$ is the vector of all the objective functions.

Box 4.1 provides a more structured summary of the ECS game.

Proposition 4.1. *The Nash equilibrium of game $\mathcal{G}$ always exists and is unique.*

Proof. Given the strict convexity of all resulting cost functions at each time step involved in (4.20), the inter-supplier ECS (4.29) is a strictly concave N-person game. Consequently, the existence of a Nash equilibrium directly results from [112, Th. 1]. Furthermore, it is necessarily unique by [112, Th. 3]. ■

Proposition 4.2. *The unique Nash equilibrium of game* (4.29) *is the optimal solution of the energy cost minimization problem* (4.20)*.*

Proof. In (4.29), Ω_n is assumed to be constant (non-elastic demand hypothesis). Therefore, the only way to increase the profit is to minimize the second term of the equation, which corresponds to the total bill across all community members. By definition, this is equivalent to solving problem (4.21). ■

4.3.4. Decentralized algorithm

As stated in Proposition 4.2, the global optimum of the energy cost minimization problem can be achieved concurrently with reaching the unique Nash equilibrium. Consequently, all algorithms addressing convex problem optimization can be used to compute the game solution. However, considering that prosumers are all equipped with smart meters enhanced with communication and processing resources, it is interesting to resort to a decentralized algorithm. It has the advantage of eliminating the need for a central authority and thus giving more autonomy to community members.

For this particular problem structure, it is possible to find the fixed point of the best-response mapping as described in 3.3.2. In particular, an asynchronous scheme, i.e., members may update their strategies at different times, is considered.

Let $\mathcal{K}_n \preceq \mathcal{K} \preceq \{0, 1, 2, \ldots\}$, be the set of times at which member n updates his schedule (strategy) Λ_n. If $k \notin \mathcal{K}_n$, then $\Lambda_n^k = \Lambda_n^{k-1}$. Furthermore, let $t_m^n(k)$, denote the most recent time at which the schedule of member m is perceived by member n at the kth iteration. The formal description of the algorithm, referred to as the asynchronous best-response algorithm, is the following:

Algorithm 1 Asynchronous Best-Response Algorithm

Data: Choose any feasible starting point Λ^0, set $k = 0$.
1: **while** a suitable termination criterion is not satisfied, **do**
2: **for** $n \in \mathcal{N}$, each user computes Λ_n^{k+1} as **do**
3:

$$\Lambda_n^{(k+1)} = \begin{cases} \Lambda_n^\star \in \underset{\Lambda_n}{\operatorname{argmin}}\, b_n(\Lambda_n, \Lambda_{-n}^{(\mathbf{t}^n(k))}), & \text{if } k \in \mathcal{K}_n \\ \Lambda_n^{(k)}, & \text{otherwise} \end{cases} \tag{4.30}$$

4: **end for**
5: $k \leftarrow k + 1$.
6: **end while**

where

$$\Lambda_{-n}^{(\mathbf{t}^n(k))} \triangleq (\Lambda_{-n}^{(t_1^n(k))}, \ldots, \Lambda_{-n}^{(t_{n-1}^n(k))}, \Lambda_{-n}^{(t_{n+1}^n(k))}, \ldots, \Lambda_{-n}^{(t_N^n(k))}) \tag{4.31}$$

The convergence and optimality of this algorithm are demonstrated in [113, Th. 4.2]. When an individual updates his schedule, i.e., $\Lambda_n^{k+1}, k \in \mathcal{K}_n$ in step 3 of the algorithm, the corresponding minimization problem is

$$\min_{\Lambda_n} \sum_{t \in \mathcal{T}} \left(\gamma^t_{\mathrm{com},i} l^{t+}_n \Delta\tau + \gamma_{\mathrm{grid}} ((L^t_{-n} + l^t_n)\Delta\tau)^2 \right) \qquad (4.32)$$
$$\text{s.t. constraints} \quad (4.9) - (4.12),$$

where $L^t_{-n} = \sum_{m \in \mathcal{N} \setminus n} l^t_m$. It can be noticed that problem (4.32) has only local variables in n.

Hence, each user solves iteratively its local problem, which is a standard quadratic optimization algorithm such as the simplex or the interior-point method (cf. 3.2.3). Then, the updated solutions are broadcast. **It can be noted that only the aggregated energy consumption vector l^t_m is exchanged between the smart meters of the community members, but it does not need to be stored and made available to any party.** This is a significant asset because by ensuring sufficient privacy, end-user participation and engagement may be increased. One other major advantage highlighted in the previous subsection is the strategy-proof property of the algorithm. Indeed, as the users are charged proportionally to the total energy cost across all members, deviating from the optimum by cheating would involve a higher bill for anyone.

4.4. Benchmark

4.4.1. Case study and load specifications

The case study is built on a projection of what could be a typical residential feeder in a European country in about a decade when technology and policies should be mature enough for enhancing such DSM strategies. On the one hand, current characteristics of the electric load are retained, but on the other hand, new likely perspectives for domestic electricity demand are incorporated. Hence, the expected electrification of the transportation sector through electric and hybrid vehicles (EVs) and the electrification of the building heating systems through electric heat pumps (HPs) are considered. These radical changes are expected to increase the share of flexible loads. We choose to emphasize what could be a modern low-voltage network feeding a new neighborhood.

The average annual electricity consumption per household in Belgium (Brussels region, cf. [114]) is estimated at around 3000 kWh when considering no electric heating systems and cooking using gas. Among domestic appliances, it is reasonable to assume that some flexibility could be consented to by dishwashers, washing machines, and dryers, i.e., flexible appliances (FAs). These account for about 25 % of the load. Besides, there are resolute commitments from governments to reach

Table 4.2.: Load specifications of the study-case

Load Types	# in feeder	Max Power ($M_{n,a}$) [kW]	Energy needs ($E_{n,a}$) [kWh]	Energy Proportion
SLPs	15	NA	6	28.5%
EVs	7	4	6	13.3%
FAs	30	1	1	9.5%
HPs	11	1.6	14	48.7%

ambitious targets in terms of electric vehicle penetration, cf. 2.4. Furthermore, electric heat pumps installation grows exponentially. It is expected to provide about 20 % of the household heating by 2030, and nearly 80 % of new houses are equipped in the Netherlands [115]. Considering that households in Belgium own on average 1.5 cars [116], [117], and the growing success of heat pumps, both presenting strong flexibility potential, we choose to incorporate these two perspectives in our case study.

Hence, based on these considerations, we assume the following mean individual load characteristics for the case study:

- 6 kWh of *non-flexible load* (e.g., lighting, TV, computer, oven): a same standardized Synthetic Load Profile (SLP) is assumed for each user. In future works, forecasts could be considered.

- 2 kWh of *flexible appliances* (e.g., dishwasher, washing machine, dryer): users are attributed a certain number of 1 kWh loads with a maximum power of 1 kW.

- 3 kWh for *electric vehicles* (average of 50 km/day with 1.35 kWh/10 km, 1.5 cars/household, and 30 % penetration).

- 10.25 kWh for *heat pumps* (average of 5000 kWh/yr and 75 % penetration).

We choose to assess our energy consumption scenarios on a low-voltage feeder of $N = 15$ users with different energy needs and constraints. In each simulation, we attributed a random combination of the different load types presented in Table 4.2 and their possible scheduling possibilities ($\delta_{n,a}$). The constraints integrate reasonable habits such as the fact that most cars are home and available for charging at night. Besides, the users are attributed randomly to 4 different energy suppliers applying distinct quarterly pricing schemes.

Note that all scenarios were implemented using the Julia programming language [118]. Optimizations were achieved with the Gurobi™ solver [119] and the computation time remained under 1s (6048 variables for the inter-supplier ECS).

Table 4.3.: Results of the simulations in terms of costs

Costs [€]	**Commodity**	**Network**	**Mean Total**
Passive demand	42.21	80.16	122.38
Flattened demand	42.12	47.80	89.92
Intra-supplier ECS	36.20	53.06	89.26
Inter-supplier ECS	37.14	47.95	85.09
TC reduction [%]	**PD to FD**	**FD to Intra**	**FD to Inter**
Mean	26.5	0.7	5.4
Minimum	23.7	-4.6	3.8
Maximum	30.2	3.4	6.6

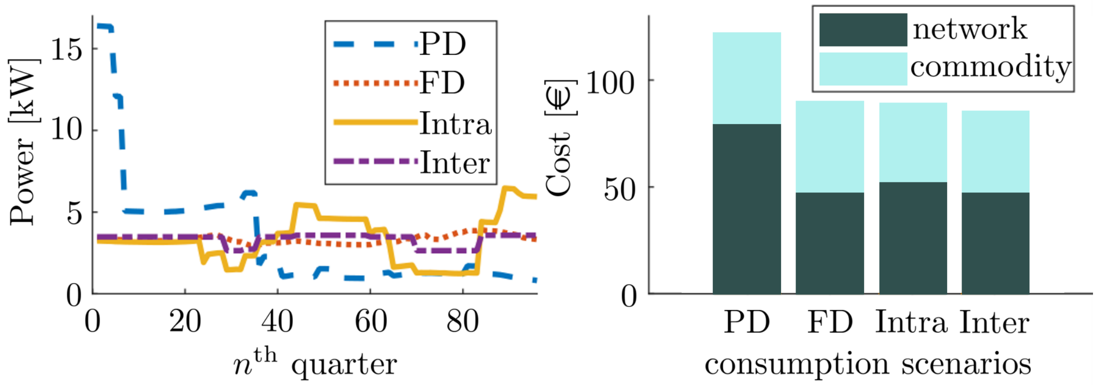


Figure 4.4.: Total power of the feeder when considering the different strategies (left) and their underlying costs (right) for one simulation example.

4.4.2. Results

Table 4.3 summarizes the outcomes after 100 simulations. Comparing the results obtained for the four different consumption strategies shows that the inter-supplier ECS scheme achieves the best results as expected. Indeed, it is the only strategy that minimizes the total cost (social optimum). Although there is a substantial decrease between passive and flattened demand strategies (PD and FD), it should be noted that the passive demand strategy is a worst-case scenario as it schedules devices with permissible time slots in the first hours of the morning all at once. In reality, it is more likely that a passive consumption behavior would lead to a somewhat more distributed schedule and thus to a situation between the passive demand and flattened demand strategies. The flattened demand strategy can be considered as an elementary load management strategy. Indeed, it requires some control capability to modulate the power consumption. However, this strategy, though oriented towards decreasing network costs, does not address the commodity costs. In contrast, intra-supplier ECS seeks the minimization of the latter costs.

Nevertheless, Table 4.3 shows that cooperating only on commodity costs fails to decrease the total costs significantly. It even turns out to be less advantageous than the flattened demand strategy in 25 % of the simulations. Indeed, as expected, the commodity contribution is greatly decreased, but the network contribution neutralizes the cost reduction. Hence, only a strategy involving a collaboration mechanism between all the users proves to be relevant. Furthermore, it does not require any additional communication as only the aggregated consumption of each user is communicated using the smart meters.

The impact of the different strategies on the aggregated power curves is depicted in Figure 4.4 through one simulation example. It shows, as previously mentioned, that the passive users have most of their consumption taking place at the beginning of the day leading to high related network costs and non-optimal commodity prices. As expected, the flattened demand leads to a curve that is much flatter. Indeed, all devices except the non-flexible loads consume a constant power over their permissible range of time. The network costs are hence significantly decreased. The intra-supplier ECS has a quite variable power curve as each user tries to exploit the cheaper time slots of their related supplier. In contrast, the inter-supplier ECS tempers the variability of the power curve to limit the increase in network costs.

The results are strongly dependent on the conditions we have imposed. Despite the effort of shaping what could be realistic future conditions, the numbers cannot be generalized. However, the main conclusions remain unchanged as long as the main hypotheses apply, i.e., reflecting as closely as possible the true costs to each actor of a liberalized electricity market by using appropriate cost functions. Whereas one individual, by the power generation requirements, affects only the other customers of its supplier, it also influences every other user on the same network through its underlying power flows. Hence, only a strategy considering an integrated approach can lead to an optimized schedule in terms of global costs. In that regard, the introduction of collaboration mechanisms proves to be judicious as it requires no services of a third party and no additional piece of equipment.

4.4.3. Impact of photovoltaic generation

In the previous section, it is shown that the inter-supplier cooperation mechanism leads to the optimal total cost solution. Despite the numerous parameters at stake, the decrease in the total bill is substantial. We show that this is further accentuated when we introduce photovoltaic generation.

Introducing solar panels in the scenarios is highly relevant. Today already, a substantial amount of solar power is deployed. Even though the increase in the share of this technology is difficult to predict due to policy uncertainties, it is clear

Table 4.4.: Results of the simulations in terms of costs; 20 % PV penetration

Mean costs [€]	**Commodity**	**Network**	**Mean Total**
Passive demand	36.28	78.24	114.52
Flattened demand	36.07	38.49	74.56
Intra-supplier ECS	30.11	41.60	71.71
Inter-supplier ECS	30.74	36.40	67.14
TC reduction [%]	**PD to FD**	**FD to Intra**	**FD to Inter**
Mean	34.9	3.8	9.9
Minimum	31.8	-6.4	7.2
Maximum	37.6	8.0	11.5

Table 4.5.: Results of the simulations in terms of costs; 47 % PV penetration

Mean costs [€]	**Commodity**	**Network**	**Mean Total**
Passive demand	27.88	87.38	115.25
Flattened demand	27.85	36.74	64.58
Intra-supplier ECS	21.96	35.86	57.82
Inter-supplier ECS	23.28	23.48	46.76
TC reduction [%]	**PD to FD**	**FD to Intra**	**FD to Inter**
Mean	43.9	10.4	27.6
Minimum	42.1	-9.7	24.7
Maximum	45.8	21.3	30.3

Table 4.6.: Results of the simulations in terms of costs; 67 % PV penetration

Mean costs [€]	**Commodity**	**Network**	**Mean Total**
Passive demand	21.95	100.13	122.08
Flattened demand	21.83	43.44	65.27
Intra-supplier ECS	15.87	39.81	55.67
Inter-supplier ECS	17.98	16.05	34.03
TC reduction [%]	**PD to FD**	**FD to Intra**	**FD to Inter**
Mean	46.5	14.6	47.9
Minimum	44.4	-16.5	45.1
Maximum	48.7	31.4	49.9

that the technical advances and the price decreases should promote the steady growth of new installations in the future.

Hence, we simulate the different strategies with the same feeder described in 4.4.1 under different PV penetration rates (20 %, 47 %, 67 %, and 100 %). One day of clear sky is reproduced with solar facilities delivering 2.88 kW each at the peak of the day.

Table 4.7.: Results of the simulations in terms of costs; 100 % PV penetration

Mean costs [€]	**Commodity**	**Network**	**Mean Total**
Passive demand	11.83	131.44	143.27
Flattened demand	11.69	65.76	77.45
Intra-supplier ECS	5.71	58.97	64.68
Inter-supplier ECS	12.07	13.74	25.81
TC reduction [%]	**PD to FD**	**FD to Intra**	**FD to Inter**
Mean	45.9	16.3	66.7
Minimum	43.2	-17.1	63.6
Maximum	48.0	41.4	69.3

The results discussed in 4.4.2 remain valid but are even more acute. Indeed, the inter-supplier ECS reduce the total costs drastically (cf. tables 4.4 - 4.7). On the other hand, a poorly effective consumption strategy such as the passive demand or flattened demand can lead to higher total costs at the higher penetration rates while a significant portion of the energy is provided by the sun for free. This paradoxical situation can be easily explained. Indeed, the network can be highly loaded if the community auto-consumption level is low, thus leading to high network costs. Optimizing only on commodity costs through the intra-supplier ECS can lead at times (14 and 5 % of the cases under 70 and 100 % penetration rate respectively) to higher prices than flattened demand for the same reasons. Indeed, it tends to favor selling the energy (we assume the same pricing scheme applied by the suppliers), thus decreasing commodity costs, but leads to high peak-to-average ratio on the network. These considerations are reflected in the shapes of the total power of the feeder and the different underlying costs obtained for one example of simulation, cf. Figure 4.5.

Introducing PV in the inter-supplier ECS scenario does not affect the distribution of the costs nor the Nash equilibrium. Indeed, the previously introduced expressions remain unchanged. The distribution is fixed and accounts for the ratio when optimization is completed only on commodity costs. Only e_n and E_i, respectively, the individual energy and total energy of supplier i, are consumed and withdrawn from the electric network. The energy produced and consumed in situ by an individual is therefore not taken into account. Henceforth, PV facilities provide free energy for its owner when available, and the surplus allows to decrease its supplier cooperative pool bill. Some additional distribution mechanisms could be examined, for instance, the retribution of the owner for their surplus at the supplier price. The recipient would still benefit from this distribution because of the decreased underlying network costs. However, it would override objective 2 of this work, which aims at reflecting the best image of the incurred "truth" costs.

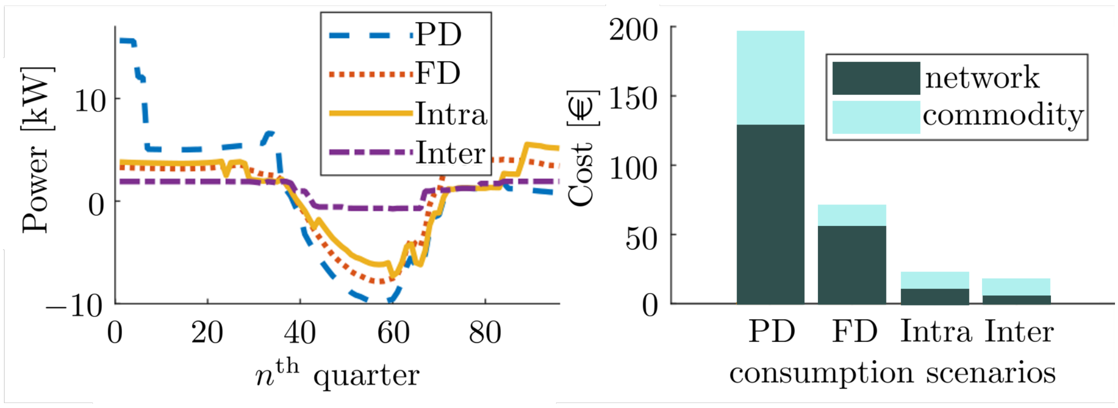


Figure 4.5.: Total power of the feeder when considering the different strategies (left) and their underlying costs (right) for one simulation example with 100 % PV penetration rate.

Indeed, photovoltaic generation has zero marginal production cost.

4.5. Conclusions

Given the environmental damages that human activity cause, minimizing the actual incurred costs (for society) should be policymakers' top priority as highlighted in Chapter 1. This is why prosumers should all be aware of the actual costs they pose. The cost structure adopted in this chapter and the following ones move in that direction. Additionally, the dynamic pricing scheme provides the main signal for time of use demand response (cf. 2.4.2). Specifically, an energy consumption scheduling scheme was established and allows benefiting from the diversity of time flexibility each user holds for their devices. End-users are incentivized to participate in the optimization process because the network costs are aggregated and then shared. In that way, all consumption actions of one end-user modify the price for all the other end-users. Such a bounding naturally enforces coordination.

This chapter has thus presented a solution for optimized day-ahead scheduling of residential consumers connected to the same network in terms of global costs, i.e., commodity and network costs. Firstly, the context and various energy consumption strategies were introduced. Then, the methodology was described, with an emphasis on the inter-supplier ECS, which is the core scenario of this chapter. The decentralized algorithm solves the optimization problem while keeping privacy. Finally, a benchmark conducted on 15 end-users of the same low-voltage network (the community) was studied. Results confirmed that in a prospective scenario, the inter-supplier cooperation strategy introduces, substantial savings on the total cost in comparison with different DSM strategies without the collaboration mechanism. The savings are much more pronounced with the introduction of PV because network flow management is even more beneficial. The optimized solution reached by the inter-supplier ECS and its inherent Nash equilibrium in the cost distribution among users proves to be an efficient option for smarter electricity consumption.

Intraday modifications of the schedule and events requiring short-time reactions as well as the errors on the forecasted non-shiftable load introduce deviations from the initial solution. Studying the impacts of such deviations and developing solutions handling these considerations is valuable. Some perspectives are highlighted in Chapter 7. In addition, developing solutions to handle the uncertainty of the non-shiftable load is also a notable prospect. It is stressed by the introduction of solar panels, which have a high stochastic behavior.

Hence, this chapter has introduced the groundwork of the original energy consumption scheme that is put forward in this thesis. The truth cost approach has led to aggregate the costs of all end-users, which, in turn, raises the crucial question of cost distribution. The provided distribution key of this chapter is based on a static daily billing. The next chapter evaluates more possibilities for the billing, of which a multi-temporal (continuous) billing. It is complemented by the addition of personal storage. Besides, although the coordination of end-users to optimize incurred costs as close as possible meets a societal goal, transactions take place only between end-users and their respective supplier. Chapter 6 introduces a framework enhancing exchanges between members, which turn the entity into an impactful community.

This chapter has led to the following publication:

M. Hupez, Z. De Grève and F. Vallée, "Cooperative demand-side management scenario for the low-voltage network in liberalised electricity markets," in *IET Generation Transmission and Distribution*, vol. 12, no. 22, pp. 5990-5999, 2018.

CHAPTER 5.

Assessing prosumer incentivization in energy communities under game-theoretical billings

The previous chapter exposed a collaborative framework of joint energy consumption scheduling in a low-voltage entity (community) that lays the groundwork of the main proposal, i.e., responsible energy communities. Particular attention was brought to define a price signal that reflects as much as possible the incurred costs. In addition, it presented a billing scheme that is in line with the liberalization. The resulting Nash equilibrium leads to optimizing the social cost. However, it is not necessarily the fairest and most incentivizing method (e.g., passive members are equally treated), which, in turn, could prevent prosumers from consenting to more flexibility.

In this chapter, three relevant billing schemes are developed and compared in a similar framework. The aim is to assess incentivization and fairness considerations on various prosumer profiles. The first two approaches rely on a (static) daily billing scheme, while the third considers a multi-temporal (continuous) billing. The Nash equilibria are computed using decentralized algorithms, hence ensuring sufficient privacy and removing third-party dependencies. The cost distributions are assessed, using both a qualitative and a quantitative comparison based on various prosumer profiles, on a modern smart grid. It is shown that depending on the billing option, either the contribution towards the entity (ability to improve the global solution) or individual empowerment (ability to bargain) can be preferentially incentivized. Hence, depending on the level of community feeling, sensitivity, and responsiveness characterizing the entity, one option or the other may better fit the purpose.

Note that this chapter's content is largely derived from the journal article "Pricing Electricity in Residential Communities using Game-Theoretical Billings", currently under review in *IEEE Transactions on Smart Grid, special section on Local and Distribution Electricity Markets* [26].

In the remainder of this chapter, the contextual elements and the state of the art are presented in Section 5.1. Section 5.2 briefly recalls the demand-side model and then details the various billing models, the underlying games, and how the solutions are computed. The following section details and discusses the results obtained under various comparatives. Finally, Section 5.4 summarizes the main findings and prospects.

5.1. Introduction

By sharing common assets such as the power grid, prosumers are closely interrelated by their actions and interests as highlighted in Chapter 3. In that context, game theory provides powerful tools for increased coordination between prosumers to optimize energy resources. However, depending on the prosumer profiles and the market rules, individual bills can notably differ and prove to be unfair or little incentivizing.

In the previous chapter, an adaptation of the energy consumption scheduling game to the reality of liberalized electricity markets was proposed. The prosumers retained the free choice of their retail supplier, and the costs of the shared low-voltage grid are accounted for by considering a billing scheme that reflects the incurred costs. However, it does not consider anything else than a daily proportional billing. It is certainly interesting to study other cost distributions. In addition, more dynamic schemes such as the continuous proportional billing, which is a multi-temporal billing scheme, are expected to better incentivize flexible scheduling.

Beside these methodological considerations, it is essential to qualify and quantify the cost allocations with respect to the prosumer profiles. Indeed, technological advances have profoundly changed the nature of the energy load and transformed end-users into full-fledged actors. Their associated flexibility and constraints heavily influence the outcomes of demand-side management programs. Currently, there is a flagrant lack of perspective on the application of one billing method or another concerning the cost distribution and the load profiles. In this respect, most works simply highlight the global performance and evaluate fairness under a single dimension. For instance, [120] examines daily and hourly billings using an elementary framework in which only flexible appliances can be mobilized (without addressing individual storage and production means) and where a single utility company can be contracted. In this way, the game solution can be easily computed using simple best-response algorithms.

In this chapter, the day-ahead scheduling of energy consumption in a residential community is studied. In particular, three billing schemes are transposed to the current liberalized context while accounting for individual generation and storage.

As in Chapter 4, special emphasis is placed on faithfully reflecting the incurred costs (commodity and grid costs). The main addition lies in combining these methods and their evaluation with a comparison regarding cost allocation between prosumers. **This chapter aims at filling the gap in exposing various incentivization means targeting different objectives, hence addressing fairness in diverse ways.** More specifically, the following contributions are provided:

i) Three different billing methods in a liberalized residential community are proposed. In such a context, each end-user has freely contracted an electricity supplier, while the grid costs are charged to the community as a whole by the transmission and distribution system operators. In this way, it differs from most of the literature, which considers that a single utility company is contracted for commodity and grid costs. The first two billing methods adopt a daily proportional billing, which considers a single distribution key for the entire time horizon (i.e., one day). Among these, one uses a distribution key that is directly proportional to the net load whereas the other one passes on the relative contribution through a Vickrey-Clarke-Groves (VCG) mechanism. Their outcomes are readily computed using the asynchronous best-response algorithm. The third billing method distributes the costs proportionally to the net load at each time slot of the scheduling horizon. For the latter, we resort to the proximal decomposition algorithm. All three billing methods incentivize prosumers differently depending on the load profiles.

ii) A comprehensive analysis of the aforementioned billing methods is provided and their performance in the realistic context of a modern and liberalized residential community is assessed. The prosumer profiles are characterized by possible storage and production means as well as electrical appliances holding significant scheduling flexibility (such as electrical vehicles and heat pumps). A qualitative comparison highlights how the prosumers are impacted depending on their load profile and the billing methods. Moreover, a benchmark featuring a representative mix of prosumer profiles based on a real database from Pecan Street[1] dataport provides a quantitative assessment.

5.2. System structure and methodology

In 4.2, the common framework and the elementary model were introduced. The power network, the prosumer load model and the electricity cost model remain substantially unchanged.

[1] Pecan Street is an organization delivering data on consumer energy and water consumption behavior, testing and verifying technology solutions, and commercializing related services.

Hence, a modern power grid where each prosumer is connected to a bi-directional communication infrastructure (using, e.g., a smart meter) is considered. A liberalized framework is assumed, i.e., the power generated by the electricity producers flows through the transmission grid and the distribution network, and is sold by an electricity supplier to the end-users, who share the same low-voltage network, e.g., behind the same low-voltage distribution transformer. These prosumers need to coordinate their energy consumption scheduling with all the other end-users because they share the same network and have interdependent costs (cf. Section 5.3.1). Together, they thus act as an embryonic community.

5.2.1. Demand-side model

The demand-side model is almost identical to what is considered in the basic scenario, cf. 4.2. Hence, the power network elements and the electricity cost and pricing model are the same. The prosumer model, however, is complemented by storage variables, parameters and constraints, but they were already defined in the elementary model, cf. 4.2.4.

The following figure (Figure 5.1) summarizes the main groups and variables.

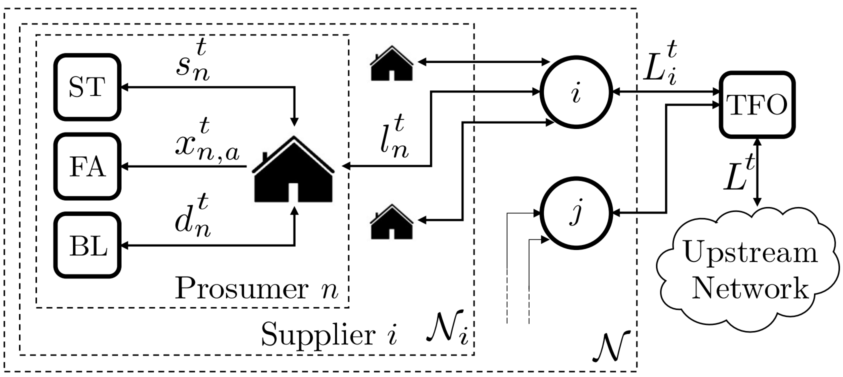


Figure 5.1.: Demand-side model: prosumer n, who contracted supplier i, owns a storage system (ST), flexible appliances (FA), and is characterized by a baseload (BL) grouping non-flexible consumption and PV generation. All the prosumers across all the suppliers behind the same transformer (TFO) form the residential community.

5.2.2. Billing models

From the cost functions in (4.15) and (4.16), it is possible to establish billing models that help to achieve specific performance objectives, where efficiency and fairness occur to be among the most commonly sought ones. In this chapter, we use and compare three different billing methods (cf. Section 5.3), each holding advantages and drawbacks regarding these objectives. They originate from the following two billing classes.

i) **Daily proportional billing.** It consists in issuing bills proportionally to the costs aggregated across both prosumers and time (on a daily basis). The cumulative cost function, unchanged from its definition in Chapter 4, is denoted by

$$f(\Lambda) = \sum_{t \in \mathcal{T}} \left(\sum_{i \in S} \gamma^t_{\text{com},i} L^{t+}_i \Delta\tau + \gamma_{\text{grid}} (L^t \Delta\tau)^2 \right), \tag{5.1}$$

where $L^{t+}_i \triangleq \sum_{n \in \mathcal{N}_i} l^{t+}_n$. This type of billing is optimal from the system's perspective in the sense that the prosumers are incentivized to minimize the cumulative cost $f(\Lambda)$, as discussed in Section 5.2.3. Under a daily proportional distribution, the billing function is given by

$$b_n(\Lambda) = \frac{w_n}{\sum_{m \in \mathcal{N}} w_m} f(\Lambda), \tag{5.2}$$

where w_n represents the weight of prosumer n in the bill. However, it is a challenging task to redistribute these costs among the prosumers by implementing a fair set of weights. Indeed, the aggregative nature of the grid costs makes it difficult to fairly identify the individual contributions. A straightforward weight is obtained by considering the cumulative net positive load of the individual, i.e., $\sum_{t \in \mathcal{T}} l^{t+}_n$. Its minimum possible value, denoted by $\mathrm{L}^\star_n \triangleq \min(\sum_{t \in \mathcal{T}} l^{t+}_n)$, is considered so as to keep the weight independent from the actual solution set Λ (for the benefit of the total cost, the algorithm could use the minimization potential of L_n of the different members more or less, which is unfair). Such a weight choice ensures that there is no possible strategy on the distribution key. The main flaw of this cost distribution is that it does not link one's bill directly to its choice of supplier (i.e., prosumers who negotiate good contracts with their supplier are not incentivized) and to its flexibility level. Another choice for the weights is obtained by considering the relative contribution of each prosumer by using the normalized VCG mechanism [106] or the Shapley value [25]. It is the first option that is developed further in this chapter because of its reduced computation needs (cf. Section 5.3.1). An interesting alternative, described in [24], considers weights that are based on the optimized cost they would get using an individual-based scheduling, i.e., without knowledge of the others' actions.

ii) **Continuous proportional billing.** It consists in issuing bills proportionally to the costs aggregated across prosumers for each time slot $t \in \mathcal{T}$, the set of optimization intervals. The cost function is denoted by

$$f^t(\Lambda^t) = \sum_{i \in S} \gamma^t_{\mathrm{com},i} L^{t+}_i \Delta\tau + \gamma_{\mathrm{grid}} (L^t \Delta\tau)^2. \tag{5.3}$$

Under a continuous proportional distribution, the billing function is given by

$$b_n(\Lambda) = \sum_{t \in \mathcal{T}} \frac{l^{t+}_n}{L^{t+}} f^t(\Lambda^t), \tag{5.4}$$

which accounts for the amount of energy exchanged by an individual and charges directly the underlying cost [107], [108], [121]–[124]. Unlike (5.2), there is possible strategy on the distribution key. **Although it does not yield the social (global) optimum, the inefficiency is expected to be more than compensated by the higher overall flexibility leading to reduced costs.** Indeed, the continuous proportional billing tends to be fairer than the daily proportional billing. Planning power exchanges at the preferential times of the dynamic pricing is better valued in the individual bills [125]. Incentivization is thus more effective.

5.2.3. Energy consumption scheduling

Due to the competition arising from the shared use of the network, it is appropriate to adopt a game-theoretical formulation to characterize potential strategic behaviors. This section introduces the game formulation and shows how the underlying Nash equilibria are computed.

Game formulation

Non-cooperative game theory, as briefly exposed in 3.3, is a powerful mathematical framework for modeling the interactions between selfish individuals competing for a common resource [86], [113]. It is thus meaningful to formulate the energy consumption scheduling problem as a non-cooperative game, independently of the chosen billing method. In this context, each prosumer $n \in \mathcal{N}$ is a player who competes against the others by choosing its strategy profile Λ_n to minimize its objective function $b_n(\Lambda)$ (defined either as in (5.2) or as in (5.4)).

Let $\Lambda_{-n} \triangleq \{\Lambda_m\}_{m \in \mathcal{N} \setminus \{n\}}$ denote the set of all the strategy profiles except those of player n. We formally define the game by the tuple $\mathcal{G} \triangleq \langle \Omega, \mathbf{b} \rangle$, where $\Omega \triangleq \prod_{n \in \mathcal{N}} \Omega_n$ is the joint strategy set, with Ω_n being the individual strategy set of prosumer n, and $\mathbf{b} \triangleq \big(b_n(\Lambda_n, \Lambda_{-n})\big)_{n \in \mathcal{N}}$ is the vector of all the objective functions. Hence, each player n aims at solving the following optimization problem, given Λ_{-n}:

$$\begin{array}{ll} \min\limits_{\Lambda_n} & b_n(\Lambda_n, \Lambda_{-n}) \\ \text{s.t.} & \Lambda_n \in \Omega_n \end{array} \qquad \forall n \in \mathcal{N}. \tag{5.5}$$

Similarly to the ECS game (4.29) in the previous chapter, the solution of $\mathcal{G}$ is given by the Nash equilibrium, i.e., finding a feasible strategy profile $\Lambda^\star \triangleq \{\Lambda_n^\star\}_{n\in\mathcal{N}}$ so that no single player n can benefit by unilaterally deviating from $\Lambda^\star$ if all the other players act according to $\Lambda_{-n}^\star \triangleq \{\Lambda_m^\star\}_{m\in\mathcal{N}\setminus\{n\}}$.

Proposition 5.1. *The game $\mathcal{G}$ has a non-empty and compact set of Nash equilibria.*

Proof: Building on [86], [113], the above is guaranteed when the following conditions hold for each player n:

i) the individual strategy set Ω_n is compact and convex;

ii) the objective function $b_n(\Lambda_n, \Lambda_{-n})$ is convex for any feasible Λ_{-n}.

The first condition is easily verified since Ω_n consists in a set of linear inequalities (4.9)–(4.14) whereas the second condition can be proved by showing that the Hessian matrix of $b_n(\Lambda)$ (defined either as in (5.2) or as in (5.4)) is positive semidefinite. ■

Proposition 5.2. *All the Nash equilibria of $\mathcal{G}$ yield the same values of the objective functions.*

Proof: Building on [107], [108], it is easy to show that there exists an infinity of strategy profiles producing the same net load vectors $\mathbf{l}_n$. ■

Hence, there exists at least one Nash equilibrium and the game always yields the same value.

Nash equilibrium computation

In the case of a daily proportional billing, if the strategy of each prosumer n is computed by minimizing the billing function (5.2) via (asynchronous) best-response algorithm (see, e.g., [101]), the cumulative cost (5.1) can either decrease or remain constant. In this setting, the Nash equilibrium is reached when no player can decrease its bill, i.e., when the cumulative cost is minimized (social optimum). Hence, in this specific case, it is possible to consider the Nash equilibrium as the solution of the system optimization problem

$$\begin{array}{ll} \min\limits_{\Lambda} & f(\Lambda) \text{ as in (5.1)} \\ \text{s.t.} & \text{(4.9)–(4.14),} \end{array} \tag{5.6}$$

which can be solved either by a centralized algorithm or by using a distributed implementation based, for instance, on the *Alternative Direction Method of Multipliers* (ADMM) [126].

On the other hand, pure *best-response algorithms* cannot be used in the case of continuous proportional billing because minimizing the billing function (5.4) for any prosumer n cannot guarantee that the cumulative costs (5.3) is not increased. However, one can use more sophisticated distributed schemes such as the *proximal decomposition algorithm* or the *proximal-point method* [127], which are guaranteed to converge under some technical conditions (the latter additionally requires strict monotonicity of the cost function). For instance, the convergence conditions for the proximal decomposition algorithm can be conveniently derived by resorting to variational inequality theory as done in [107], [108], [121] (see also [86], [113]).

Without loss of generality, we adopt the asynchronous best-response algorithm (cf. Algorithm 2) in the case of daily proportional billing and the proximal decomposition algorithm (see Algorithm 3) in the case of continuous proportional billing. The latter is based on the regularized game

$$\begin{array}{ll} \min\limits_{\Lambda_n} & b_n(\Lambda_n, \Lambda_{-n}) + \dfrac{\kappa}{2}\|\Lambda_n - \Lambda_n^{(i)}\|^2 \\ \text{s.t.} & \Lambda_n \in \Omega_n \end{array} \quad \forall n \in \mathcal{N}, \tag{5.7}$$

which, for a sufficiently large regularization parameter $\kappa > 0$ has a unique solution that can be computed in the same way than the best-response algorithm (cf. Algorithm 2).

These schemes have desirable privacy-preserving properties since only the aggregate load is necessary for each prosumer to compute its intermediary solution. Furthermore, the distributed feature of the algorithms does not require the intervention of a third party.

The mathematical proofs of the algorithms and conditions for their convergence to one of the Nash equilibria, are beyond the scope of this contribution. We refer to [101], [107], [108], [121] for more details.

5.3. Comparative study

As a basis for our different test cases, we use residential consumption data from the Pecan Street dataport. The dataset covers 6 months of electricity consumption and photovoltaic generation of 25 homes in the state of New York, USA.

Algorithm 2 Asynchronous Best-Response Algorithm

Data: Choose any feasible starting point Λ^0, set $i = 0$.

while a suitable termination criterion is not satisfied, **do**
  for $n \in \mathcal{N}$, each user computes Λ_n^{i+1} as **do**

$$\Lambda_n^{(i+1)} = \begin{cases} \Lambda_n^\star \in \underset{\Lambda_n}{\operatorname{argmin}}\, b_n(\Lambda_n, \Lambda_{-n}^{(t^n(i))}), & \text{if } i \in \mathcal{T}_n \\ \Lambda_n^{(i)}, & \text{otherwise} \end{cases} \tag{5.8}$$

  end for
  $i \leftarrow i + 1$.
end while

Algorithm 3 Proximal Decomposition Algorithm

Data: Set $i = 0$ and the initial centroid $\bar{\Lambda}^{(0)} = \mathbf{0}$. Choose any feasible starting point $\Lambda^{(0)}$, and $\kappa > 0$.

while a suitable termination criterion is not satisfied, **do**
  for $n \in \mathcal{N}$, each user computes Λ_n^{i+1} as **do**

$$\Lambda_n^{i+1} \in \underset{\Lambda_n}{\operatorname{argmin}} \{ b_n(\Lambda_n, \Lambda_{-n}^i) + \frac{\kappa}{2} \|\Lambda_n - \bar{\Lambda}_n\|^2 \} \tag{5.9}$$

  end for
  if the NE has been reached **then**
    each user $n \in \mathcal{N}$ updates his centroid x_n:
    $\bar{\Lambda}_n = \Lambda_n^{(i+1)}$.
  end if
  $i \leftarrow i + 1$.
end while

5.3.1. Billing and prosumer loads

In our comparative study, we use the following cost distribution schemes, each holding different interesting properties.

i) **Net load proportional billing [Net].** We use a variant of the daily proportional billing that distributes the bill proportionally to the minimum possible cumulative positive net load. **Opting for the minimum possible cumulative positive net load instead of the actual cumulative positive net load ensures that an individual cannot be penalized for contributing to a better social cost, hence avoiding strategic behaviors.** For example, an individual may be asked to inject its PV generation on the network instead of storing the energy for personal future use. This may occur to prevent load peaks on the grid and decrease the overall bill. Hence, we choose each weight such that $w_n = \mathrm{L}_n^\star$, which yields (cf. (5.2))

$$b_n^{\mathrm{Net}} = \frac{\mathrm{L}_n^\star}{\sum_{m=1}^{N} \mathrm{L}_m^\star} f(\Lambda). \tag{5.10}$$

Note that accounting for the positive net load encourages the self-consumption of locally produced electricity. This is in line with the cost structure that assumes no purchasing price for the injection. **This billing scheme is equivalent to the inter-supplier scenario presented in 4.3.3. However, individual storage is included in this chapter (cf. Table 4.1).**

ii) **Marginal cost billing [VCG].** We use another variant of the daily proportional billing that is shown to be the only VCG mechanism ensuring a dominant truthful strategy [128]. Here, we choose each weight such that $w_n = \mathrm{C}_{\mathcal{N}}^\star - \mathrm{C}_{\mathcal{N}\setminus\{n\}}^\star$, where $\mathrm{C}_{\mathcal{N}}^\star$ is the optimal cost achieved by the whole set of prosumers $\mathcal{N}$ by solving (5.6), which yields (cf. (5.2))

$$b_n^{\mathrm{VCG}} = \frac{\mathrm{C}_{\mathcal{N}}^\star - \mathrm{C}_{\mathcal{N}\setminus\{n\}}^\star}{\sum_{m\in\mathcal{N}} \mathrm{C}_{\mathcal{N}}^\star - \mathrm{C}_{\mathcal{N}\setminus\{m\}}^\star} f(\Lambda). \tag{5.11}$$

This scheme requires the additional solution of N optimization problems to obtain each $\mathrm{C}_{\mathcal{N}\setminus\{n\}}^\star$. This can be conveniently done in a distributed fashion via the ADMM, as in the sharing problem presented in [126]. Note that the marginal cost pricing can also be formalized as a coalitional game. Indeed, in such a VCG mechanism, a predefined consensus can be reached, and the maximum value (i.e., savings) is achieved when the grand coalition (i.e., the whole community) collaborates.

iii) **Continuous proportional billing [CP].** We use a variant of (5.4) that integrates the cost considerations inherent to a liberalized context, which yields

$$b_n^{\mathrm{CP}} = \sum_{t \in \mathcal{T}} (\gamma_{\mathrm{com},i}^t l_n^{t+} \Delta\tau + \gamma_{\mathrm{grid}} l_n^t L^t (\Delta\tau)^2). \tag{5.12}$$

Unlike the first two methods, this billing accounts directly for the contracted supplier through $\gamma_{\mathrm{com},i}^t$, cf. (4.15). In this context, we use the proximal decomposition algorithm to ensure convergence to a Nash equilibrium.

In addition, for each of the scenarios described above, the prosumers can own one or more of the following devices.

- *Photovoltaic facility (PV).* It is considered as a negative non-flexible load.
- *Energy storage system (ST).* It can store possible individual surplus production from the PV or store energy during cheaper hours.
- *Electric vehicle (EV).* Two time windows (morning and evening) are made available for charging, each with its energy constraint. It is thus considered as a partially flexible load.
- *Heat pump (HP).* For simplicity, and without loss of generality, it is considered as a fully flexible load.

The parameters considered for each of the devices are based on modern specifications and can be found in [129].

Note that all scenarios were implemented with Julia. Optimizations were achieved with the Gurobi™ solver and the iterative algorithms all converged within 10s.

5.3.2. Profiles and cost allocation

In this section, we aim at characterizing how the nature of the load impacts the allocation of the electricity bill. Based on the flexible devices detailed previously (i.e., PV, ST, EV, and HP), we define four different qualitative scenarios and compare their billing outcomes for each billing scheme. **Each scenario considers a reference *consumer*, whose baseload corresponds to the average load over the dataset, and a *prosumer* with the same baseload who additionally owns one or more flexible devices.** Both are assumed to have contracted a supplier that offers a cheaper night rate for the commodity. Note that this setting aims at providing a generic and simple comparison whereas a more representative and detailed benchmark is presented in 5.3.3.

Hence, Figure 5.2 considers the scenarios where the prosumer is equipped with: (a) PV, (b) EV+HP+ST, (c) PV+EV+HP+ST, and (d) PV+EV+HP+ST. Each

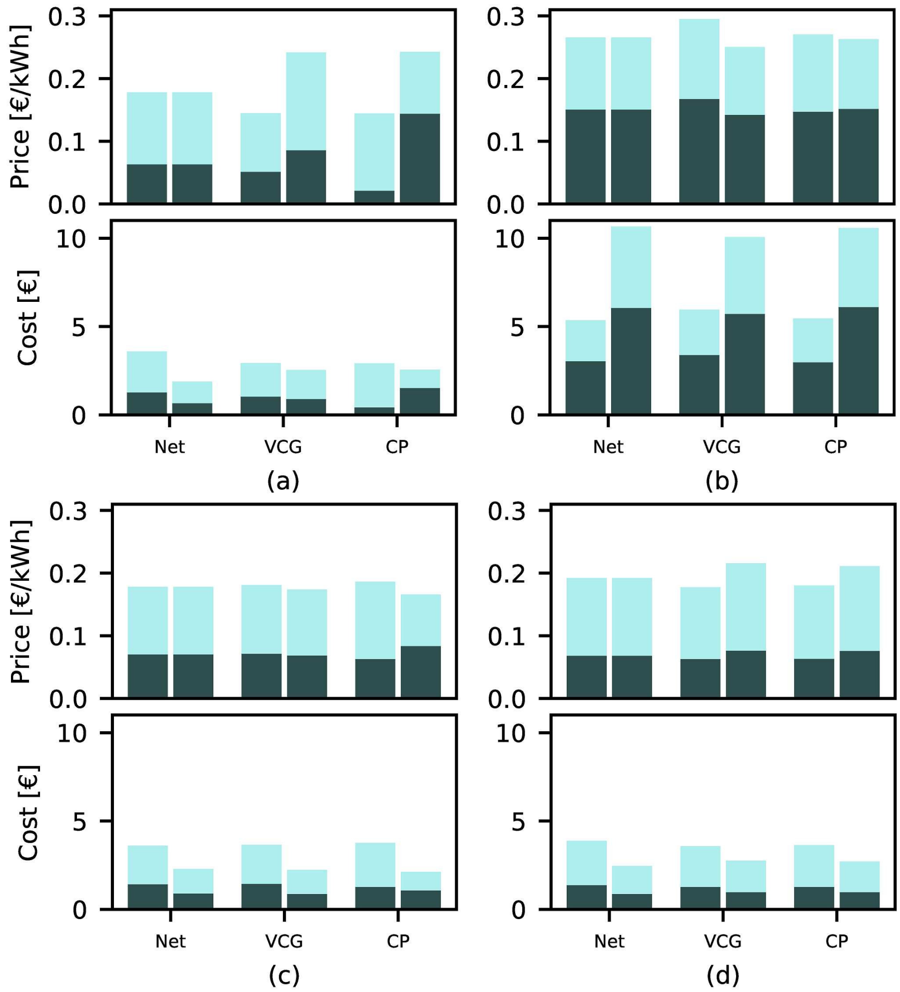


Figure 5.2.: Comparison of the prices (top figures) and costs (bottom figures) for the three considered billings, i.e., b_n^{Net} in (5.10), b_n^{VCG} in (5.11), and b_n^{CP} in (5.12). Each subplot considers a reference consumer (left bar) and a prosumer (right bar) equipped with: (a) PV, (b) EV+HP+ST, (c) PV+EV+HP+ST, and (d) PV+EV+HP+ST (with different supplier). The prices and costs are divided into grid components (dark color) and commodity components (light color).

subplot shows the different billing schemes in the x-axis and the prices or costs in the y-axis obtained by the consumer (left bar) and the prosumer (right bar). Here, the prices are obtained as the division of the cost by the respective minimum positive net load $\mathrm{L}_n^{\star}$. Furthermore, we explicitly show the prices and costs associated with the grid components (dark color) and the commodity components (light color).

Figure 5.2(a) depicts the outcome of a scenario with no flexible loads and for which the prosumer is distinguished only by the presence of a PV system generating the average production obtained on the dataset. As expected, the net load proportional billing b_n^{Net} in (5.10) features the same price for both the consumer and the prosumer, which in turn leads to a significant cost reduction for the PV holder because of the reduced expected net load. This cost reduction is less pronounced for the other two (discriminative) cost distributions and their incurred price is therefore higher. Note that such a higher price originates from an increase in both commodity and grid prices for the marginal cost billing b_n^{VCG} in (5.11), whereas it is due only to an increased grid price for the continuous proportional billing b_n^{CP} in (5.12). The latter billing scheme represents the only cost distribution where the commodity and grid costs are individually computed. By consuming little energy from imports during the day and benefiting from cheaper energy at night, the prosumer can directly assert a claim on the reduction of the commodity costs. However, it is penalized in terms of grid costs by the overall higher impact on the network. **In general, the absence of flexibility impacts negatively the outcomes of PV holders under b_n^{VCG} and b_n^{CP}.**

In Figure 5.2(b), the prosumer holds flexible devices, namely an EV, a HP, and an ST (but no PV), and **the price obtained with the discriminative billings is lower than the one applied to the reference consumer**. This is expected because the flexibility of these DERs, which help to avoid consumption peaks, is valued through a lower price. Since the cost function is increasing and convex, the prices and the costs are all higher than in the previous scenario because of the increased load.

In Figure 5.2(c), PV and flexible loads are combined and the resulting total cost is notably reduced. This is even more evident for the prosumer as its energy needs are partially covered by the PV. Indeed, **the discriminative billings reward the efforts of the prosumer by producing lower prices for its remaining imports.** As previously discussed, the continuous proportional billing directly reflects the higher impact of the prosumer on the reduction of the commodity costs and the increase of the grid costs.

Lastly, Figure 5.2(d) illustrates the major role played by the choice of the supplier. Here, we assume the same setting as in Figure 5.2(c) except that a supplier offering cheaper day rates is chosen instead of one offering cheaper night rates. Indeed, while the choice of Figure 5.2(c) leads to cheaper prices for the prosumer, the choice of Figure 5.2(d) results in increased prices (cheap commodity price: 0.08 €/kWh, normal commodity price: 0.16 €/kWh).

5.3.3. Case-study

For a more quantitative and representative assessment of what the considered billing schemes can yield for a modern smart grid, we present the following test study considering a combination of profiles. These are based on various projections of the expected load characteristics of a modern residential community within a decade. We consider 50 prosumers corresponding to residential homes whose loads are sampled from the dataset [129]. Here, each prosumer can be equipped with PV, ST, EV, and HP. Besides, they are assigned to a supplier offering either day or night cheaper rates. The respective 20 days of highest and lowest PV production are simulated (sunny days and cloudy days). The mean prices obtained through the global optimization (i.e., $\overline{\gamma} = \mathrm{C}^{\star}_{\mathcal{N}} / \sum_{t \in \mathcal{T}} L^t$) are respectively 15.0 c€/kWh and 22.4 c€/kWh.

[%]	PV	HP	ST	All	Day rate
Day	High PV (15.0 c€/kWh)				
VCG	0.2	-7.0	-11.3	-8.5	-2.6
CP	-4.6	-6.1	-6.7	-3.9	0.7
Day	Low PV (22.4 c€/kWh)				
VCG	1.6	-0.5	0.8	0.6	-3.1
CP	0.6	-1.5	-0.2	-0.4	-2.1

Table 5.1.: Variation of the mean energy price in function of the load characteristics and suppliers for a sunny day (high PV) and a cloudy day (low PV).

For the days of highest and lowest PV production, Table 5.1 summarizes the variation of the mean price obtained depending on their profile. For instance, the first column compares the difference between the mean prices obtained by the prosumers with a PV ($\overline{\gamma}_{\mathrm{PV}}$) and without a PV ($\overline{\gamma}_{\mathrm{noPV}}$), i.e., $(\overline{\gamma}_{\mathrm{PV}} - \overline{\gamma}_{\mathrm{noPV}})/\overline{\gamma}_{\mathrm{noPV}}$. Likewise, the remaining columns show, respectively, the variation depending on whether the prosumers do or do not own an HP, an ST, or all these devices. The difference between the prosumers who contracted a supplier offering cheaper day or night rates is also shown. We recall that the prices obtained with a net load proportional billing remain equal regardless of the conditions.

The most striking observation is the considerable impact of PV production. **For the days of little PV production, the billing schemes are very weakly discriminating.** This is expected as the potential extra load tends to increase the grid costs because of higher imports, even though it can be scheduled at the most opportune time (cheaper commodity costs). It is only by contracting the supplier with cheaper day rates that the prosumers can obtain better mean prices. They are indeed less numerous (cf. [26]) and tend to cancel out the negative impact on

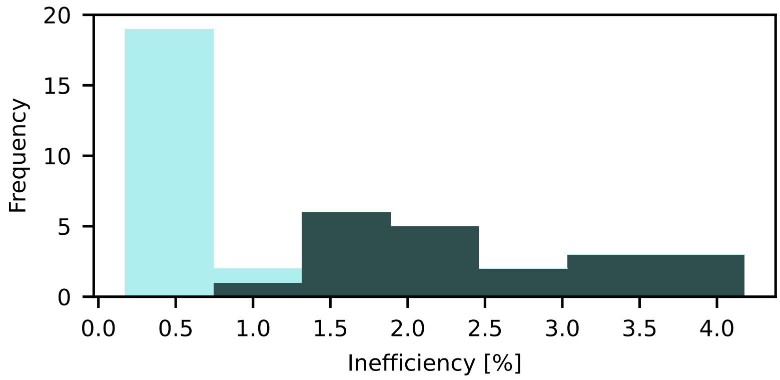


Figure 5.3.: Histogram of the inefficiency for the 20 days of respective lower PV production (light color) and higher PV production (dark color).

the grid costs caused by the preferred scheduling of flexible appliances during night hours.

The discriminative billing schemes exhibit their characteristics mostly for the days of high PV production. **The marginal cost billing (VCG mechanism) rewards the flexibility mobilized by the prosumers as a whole.** Indeed, flexible loads and storage allow to shave off the backflows that would be generated by excess PV production, hence resulting in significant grid costs savings. Interestingly, the prosumers equipped with PV benefit from lower mean prices with respect to their counterpart only if continuous proportional billing is considered. In fact, through a VCG mechanism, the PV holders cannot claim any contribution in the decrease of the total bill (as PV on its own is not a source of flexibility) whereas **the continuous proportional billing gives more space for strategy and leads to some inefficiency.**

As the continuous proportional billing yields a suboptimal solution, it is interesting to quantify the inefficiency expressed as $(\sum_{n\in\mathcal{N}} b_n^{\mathrm{CP}} - \mathrm{C}_{\mathcal{N}}^{\star})/\mathrm{C}_{\mathcal{N}}^{\star}$. For the same 20 days of highest and lowest PV production considered above, the histogram of the inefficiency is depicted in Figure 5.3. The right tail of the histogram is obtained with days of high PV production (dark color) whereas the days of low PV production lead to very little inefficiency (light color). This is further verified by considering no PV production at all under the same conditions: in that case, the inefficiency is smaller than 0.01%. The reason for the decreased efficiency during days of higher PV production is that the grid component of (5.12) is subject to strategy. For instance, the PV owners, in comparison with a socially optimal billing, tend to increase their self-consumption by just the right amount in order not to cancel the aggregate net load and remain positive. By doing so, they benefit from a payment (negative cost) because of their individual negative net load. **The days of higher PV production thus give more bargaining power to the PV**

	Efficiency	Empowerment	Tractability
Net	+	Neutral	++
VCG	+	+	Neutral
CP	Neutral	++	+

Table 5.2.: Summary of advantages (+), strong advantages (++), and neutral concerns.

owners. Note that the mean daily inefficiency for the entire dataset is 1.97%.

5.3.4. Fairness, discrimination and discussion

VCG mechanisms, when used for distributing costs, are often presented as intrinsically fair [106], [124]. In reality, this claim is subjective and highly dependent on the context and the adopted framework.

The marginal cost billing, although ensuring a truthful dominant strategy, does not give much decision-making power. Indeed, the social optimum is imposed and mobilizes more or less the resources of each individual without any further consideration. This may be little stimulating for the prosumers to consent to maximal flexibility. On the other hand, **the continuous proportional billing gives direct negotiation power to individuals**, which leads to a deviation from the social optimum. However, it is reasonable to assume that such a billing would generate higher flexibility levels, which would, in turn, more than compensate for the inherent inefficiency.

Another issue is that **although the true cost philosophy is preserved at the aggregated level, it is not the case at the individual level** where some prosumers can benefit from payments while their marginal cost is zero. An interesting framework adaptation, in line with the goal of optimizing the use of physical resources (cf. Section 5.1) consists in considering the excess of PV production as free energy made available locally to other individuals of a defined community [26]. This is the purpose of Chapter 6. In this context, PV owners are regarded as positive contributors whereas recipients are regarded as having a neutral impact although they benefit from reducing their overall demand.

On another note, **the net load proportional billing does not discriminate against individuals at all**, which might lead to little incentivization of flexibility.

Table 5.2 summarizes the important considerations highlighted throughout this chapter for each billing method. As shown in 5.3.3, the continuous proportional billing leads to very little inefficiency compared to the system optimum, which is featured by the two other methods. The marginal cost billing appears to be the billing with the best potential for mobilizing one's efforts, which justifies the ++ in the column "Empowerment". On the other hand, the flexibility is poorly incentivized under a net proportional billing, which is, however, the least demanding solution in terms of computational complexity. The marginal cost billing requires a significantly higher computational burden because it needs to solve $N+1$ optimization problems, while the decentralized algorithms of the continuous proportional billing need more attention for their convergence. Recall that all the decentralized implementations have the advantage to enforce privacy by requiring to share only the aggregate load.

5.4. Conclusions

Acknowledging the modern context of liberalized electricity networks and increased penetration of DERs, this chapter proposed and analyzed three different Nash equilibrium-based billing methods for the day-ahead scheduling of flexible appliances in a residential community. The applied cost structure that is used for billing the prosumers intends to reflect an accurate image of the mobilized energy resources. the game formulation for each billing method, namely, the net proportional billing, the marginal cost billing, and the continuous proportional billings were presented. They can be solved using practical decentralized algorithms. Then, the results obtained from solving the games allowed to derive qualitative and quantitative considerations highlighting various efficiency and fairness considerations.

The net load proportional and marginal cost billings are both socially efficient as they distribute the optimal aggregate cost of the whole low-voltage entity (community). The first does not discriminate prosumers according to their load profile, which could be perceived as little stimulating for consenting to a significant level of flexibility, whereas the second is sensitive to the cost structure and the characteristics of the individual load. The continuous proportional billing, although not socially optimal, shows very little inefficiency. Besides, it accounts for their waiver of empowerment at the benefit of the global solution.

The net load proportional billing is the most straightforward cost distribution. Although it has very limited incentive to consent flexibility, it has the advantage to be the most egalitarian. The marginal cost billing is highly relevant when the effective contribution towards the entity is promoted. Under such a scheme, the framework would be more relevant if collaboration is taken a step further by considering community resources (e.g., mutualization of excess resources as in Chapter 6) so as to have more representative distributions. On the other hand, the continuous proportional billing represents a good trade-off between higher coordination needs and self-determination.

This chapter is a significant complement to the basic framework developed previously. Whereas the latter highlighted only a daily proportional billing without accounting for possible storage capabilities, this one has developed additional billings, among which one adopted a multi-temporal cost distribution. Hence, different options incentivizing different behaviors were assessed and discussed in terms of fairness and efficiency. However, it is premature to speak about responsible energy communities in that state. To achieve this end, exchanges between end-users originating from mutualization will be introduced in the next chapter.

This chapter is an adaptation of the following publication, currently under review:

M. Hupez, J-F. Toubeau, I. Atzeni, Z. De Grève and F. Vallée, "Pricing Electricity in Residential Communities using Game-Theoretical Billings," in *IEEE Transactions on Smart Grid, special section on Local and Distribution Electricity Markets*, under review.

Part III.

Mutualization of Excesses

CHAPTER 6.

Mutualizing resources and forming responsible energy communities

The previous chapters exposed introductory aspects of responsible energy communities. The framework that was developed emphasized the necessity to account for a proper image of the incurred costs. To this end, it was shown that aggregating some costs was necessary. Hence, the network costs, which are prevailing in the total bill, are systematically aggregated because the stress applied on the network (e.g., losses and wear) results from its joint use by end-users. Besides, a concerted energy consumption scheduling scheme was presented to mobilize the time flexibility in a coordinated and fair manner, thus decreasing the total bill.

This chapter takes cooperation a step further. By enabling mutualization, the energy collectives are turned into responsible energy communities. Mutualization is applied to all excess resources inside the community. The resources made available are then attributed in order to minimize the total bill.

Note that this chapter's content is largely derived from the journal article "A New Cooperative Framework for a Fair and Cost-Optimal Allocation of Resources within a Low Voltage Electricity Community" published in *IEEE Transactions on Smart grid* (2021) [26].

The following section introduces the context and the contributions of this chapter in more detail and highlights the relevance of mutualization. Then, the community model, which builds on the previously developed models in 4 and 5, is described in Section 6.2. In particular, the load, network and cost models are successively introduced. The mechanisms of the proposed design, i.e. the optimization problem and the cost distributions, are presented in Section 6.3. The proposed scheme is examined under different case studies, and various cost distributions are compared in Section 6.4. Section 6.5 comments the implementation of continuous proportional billings and the consequences on the nature of the problem. Finally, the conclusions are drawn, and some prospects are provided.

6.1. Introduction

Two main shortcomings of the current literature on energy collectives were previously identified. Firstly, current works focused mainly on commodity costs, and neglected network-related fees, while the latter contribution represents considerable expenditures within the electricity bill. Secondly, they failed to consider that the local generation and flexibility can be valued at another price than the utility costs within the community (e.g., the truth marginal cost of photovoltaic generation is near zero), which may limit the mutualization potential of the available resources.

In this context, this thesis has proposed an original day-ahead cooperative framework aiming to minimize both commodity and network costs in a low-voltage entity, without resorting to a complex market structure. A fair distribution of the cost savings has been targeted to leverage the involvement of all its members and enhance the sharing of DERs.

In this chapter, we introduce a novel cooperation mechanism between end-users by mutualizing their excesses of resources, namely the excess photovoltaic generation and the excess storage space, and distribute them for free. By enforcing such enlightened cooperation behavior, end-users turn into responsible members. Moreover, this mechanism is indirectly advantageous for all the entity. Considering those virtuous interactions taking place between responsible and proactive members, the energy collective framework is surpassed. Accordingly to the description that was made in Chapter 2 and the objectives of this thesis, the following collectives are referred to responsible energy communities.

In addition, two different distribution keys are compared, namely a natural consensus (Nash equilibrium) arising from an individual optimum proportional billing (a daily proportional billing), and a solution that uses the Shapley value (closely related to marginal cost pricing). The proposed framework fits naturally to a distributed resolution such as the alternating direction method of multipliers [126], hence ensuring that no third-party is necessary for the reliable operation of the proposed design. The transparent computation and supervision (required to guarantee the safety and effectiveness of the proposed design) could be given by a blockchain and smart contracts [130], such that the proposed concept naturally conforms within the current liberalized framework. Overall, the main contributions of this chapter are the following.

i) A community framework where DERs and flexible resources are naturally mutualized by recording virtual power flows is established. The excess of generation and storage pertaining to individual end-users is thus made available to the rest of the community. This solution complies with the current liberalized context and raises engagement opportunities at the local level, thereby improving the cost-effectiveness of traditional strategies where each individual only considers its own electricity.

ii) A day-ahead power exchange planning that minimizes the global community costs is formulated. The procedure accounts for grid fees and losses through the Second Order Cone Programming (SOCP) relaxation of power flow equations (DistFlow). This also allows for the enforcement of a scheduling solution that complies with the network's technical requirements. Furthermore, by considering the non-linear and aggregative nature of the grid costs, it is acknowledged that the grid is a shared asset whose operation can be optimized through cooperation within the community.

iii) Two different cost distributions are applied to ensure the fairness in the final electricity bill of end-users. The goal is to allocate among individuals the costs savings from the mutualized flexibility in a transparent and rational manner. In particular, outcomes from the case study show that the consensus given by the individual optimum proportional billing tends to value the potential flexibility consented by end-users, while the Shapley value better rewards the actual mobilized flexibility.

This combination of a cost structure reflecting more accurately the use of resources with a framework enhancing higher engagement potential and a fair cost allocation to ensure the achievement of the scheme gives a holistic and innovative solution to help fulfill the energy transition objectives on the demand-side.

6.2. Community model

In this section, the contextual elements described in the elementary model 4.2 serve as starting point. Hence many variables, parameters and functions are shared with the scenarios of chapters 4 and 5 but the following paragraphs provide the additional elements that enrich this scenario.

6.2.1. Prosumer load model

To enable more efficient power exchanges through mutualization of all individual resources, **the electricity flows are differentiated between virtual and physical flows** (Figure 6.1). Virtual flows l_n^t record the energy transactions of the prosumers for billing purposes, and do not necessarily reflect the actual physical flows within the network. Obviously, the sum of all virtual and physical flows are equal.

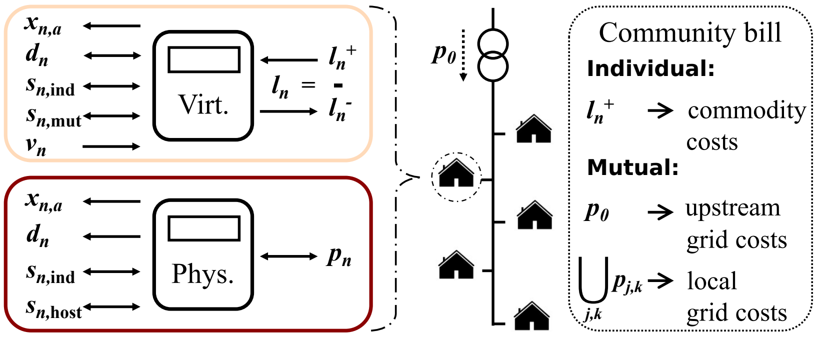


Figure 6.1.: Community bill formation based on virtual and physical flows.

In comparison with the prosumer model presented in 4.2, the storage variables are multiplied.

Hence, prosumers can have a personal storage system (ESS) such as a battery. The power scheduling vector $\mathbf{s}_{n,\text{ind}}$ for their own utilization is:

$$\mathbf{s}_{n,\text{ind}} \triangleq \left[s_{n,\text{ind}}^1, \dots, s_{n,\text{ind}}^T\right], \tag{6.1}$$

where a positive value stands for the charging mode. Then, each member makes its excess storage space available to other members in need. The power hosted $\mathbf{s}_{n,\text{host}}$ is defined by:

$$\mathbf{s}_{n,\text{host}} \triangleq \left[s_{n,\text{host}}^1, \dots, s_{n,\text{host}}^T\right]. \tag{6.2}$$

Therefore, the end-users can possibly benefit from the excess of storage space pooled by the other members of the community. The corresponding power scheduling vector $\mathbf{s}_{n,\text{mut}}$ of shared ESS space allocated to member n is:

$$\mathbf{s}_{n,\text{mut}} \triangleq \left[s_{n,\text{mut}}^1, \dots, s_{n,\text{mut}}^T\right]. \tag{6.3}$$

Similarly, the excess of power generated (i.e., outflows of end-users) within the community is made available to other members. The portion allocated to member n is given by:

$$\mathbf{v}_n \triangleq \left[v_n^1, \dots, v_n^T\right] \succeq 0. \tag{6.4}$$

Hence, the individual virtual net load (which is referred to as net load in the remainder of the chapter) metered by the DSO at time step t (Figure 6.1) is defined as:

$$l_n^t = d_n^t + \sum_{a \in \mathcal{A}_n} x_{n,a}^t + s_{n,\text{ind}}^t - v_n^t + s_{n,\text{mut}}^t. \tag{6.5}$$

As described in 4.2.4, the simplified ESS model is characterized by a maximum energy capacity E_{st} as well as maximum power levels in charge M_n^{ch} and discharge M_n^{dis} modes. They apply to both the energy stored for personal use $s_{n,\text{ind}}$ and the hosted energy $s_{n,\text{mut}}$,

$$0 \preceq \mathbf{e}_{n,\text{ind}} + \mathbf{e}_{n,\text{host}} \preceq E_{\text{st}} \tag{6.6}$$
$$-M_n^{\text{dis}} \preceq \mathbf{s}_{n,\text{ind}} + \mathbf{s}_{n,\text{host}} \preceq M_n^{\text{ch}}, \tag{6.7}$$

where $\mathbf{e}_{n,\text{ind}} \triangleq \left[e_{n,\text{ind}}^1, \ldots, e_{n,\text{ind}}^T\right]$ is the available stored energy for personal use and $\mathbf{e}_{n,\text{host}} \triangleq \left[e_{n,\text{host}}^1, \ldots, e_{n,\text{host}}^T\right]$ the available stored energy on behalf of other members, whose elements are respectively obtained by

$$e_{n,\text{ind}}^t = e_{n,\text{ind}}^{t-1} + \Delta\tau s_{n,\text{ind}}^t; \quad e_{n,\text{ind}}^t \geq 0 \quad \forall t \in \mathcal{T} \tag{6.8}$$
$$e_{n,\text{host}}^t = e_{n,\text{host}}^{t-1} + \Delta\tau s_{n,\text{host}}^t; \quad e_{n,\text{host}}^t \geq 0 \quad \forall t \in \mathcal{T}. \tag{6.9}$$

The sum of the two vectors, $\mathbf{e}_{n,\text{ind}} + \mathbf{e}_{n,\text{host}} = \mathbf{e}_{n,\text{tot}}$, hence defines the actual ESS state of charge. Additionally, the sum of charging power vectors attributed from the pool of excess storage space to each member, $s_{n,\text{mut}}$, must equal the sum of the hosting charges in the batteries $s_{n,\text{host}}$, i.e.,

$$\sum_{n \in \mathcal{N}} \mathbf{s}_{n,\text{mut}} = \sum_{n \in \mathcal{N}} \mathbf{s}_{n,\text{host}}. \tag{6.10}$$

Indeed, whereas the first sum represents a pool of virtual storage power in the eyes of the members in need, the second traces the actual physical flows of the contributing members in excess. Note that the attributed shared ESS energy $\mathbf{e}_{n,\text{mut}} \triangleq \left[e_{n,\text{mut}}^1, \ldots, e_{n,\text{mut}}^T\right]$ is obtained by:

$$e_{n,\text{mut}}^t = e_{n,\text{mut}}^{t-1} + \Delta\tau s_{n,\text{mut}}^t; \quad e_{n,\text{mut}}^t \geq 0 \quad \forall t \in \mathcal{T}. \tag{6.11}$$

Similarly, constraint (6.12) limits the available shared power to the total outflows of the members (negative net flows):

$$\sum_{n \in \mathcal{N}} v_n^t \leq \sum_{n \in \mathcal{N}} l_n^{t-}, \qquad \forall t \in \mathcal{T}, \tag{6.12}$$

where $l_n^{t-} = \max(-l_n^t, 0)$ is the negative part of l_n^t.

Finally, the excess of generated power is available only for the members who have

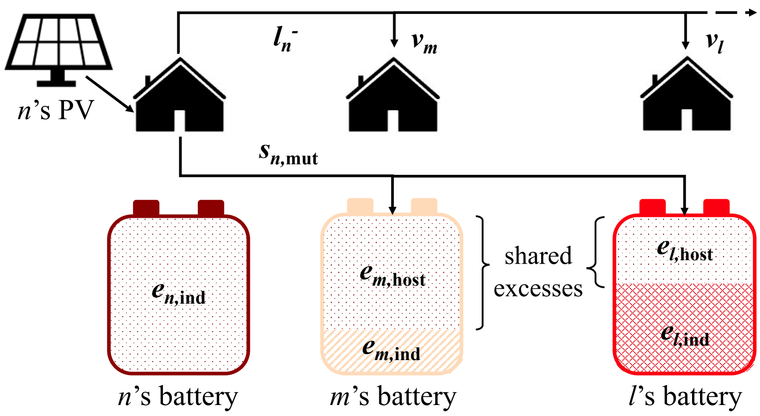


Figure 6.2.: Summary example of the variables involved in excess resources mutualization.

a positive net load, i.e.:

$$v_n^t l_n^{t-} = 0, \qquad \forall t \in \mathcal{T}. \tag{6.13}$$

A summary example is provided in Box 6.1 to illustrate the practical interest of considering virtual energy exchanges.

Box 6.1: Summary example of virtual flows

Let's consider a low-voltage feeder connecting different end-users, among which member n owns both a battery and a PV system (cf. Figure 6.2). It is assumed that its production exceeds its consumption. At a given time step (typically, after mid-afternoon), it may happen that the local production exceeds the consumption, while the storage space $e_{n,\text{ind}}$ is full (from the energy absorbed in previous hours). In traditional market designs, the energy surplus ($s_{n,\text{mut}}\Delta t$) is either consumed by local loads or transmitted outside the community. However, in the proposed framework, the energy can be stored in the batteries of other members (m and l in this example). In turn, these consider that stored energy respectively as $e_{m,\text{host}}$ and $e_{l,\text{host}}$. Although they coincide to physical flows for members m and l, they are transparent for their billing as only the virtual flows intervene as detailed in Section 6.2.3. The rest of the surplus corresponds to a negative virtual net load l_n^- that benefits partly to the other members (v_m and v_l) if they have a positive virtual net load. This framework therefore improves the value of local generation and flexibility at the community level.

6.2.2. Power network model

In order to include the costs associated to power exchanges inside the community as well as to comply with the related physical constraints, the optimal power flow equations of the radial low-voltage network are considered. In this work we adapt the DistFlow equations of [131] to a Second Order Cone Programming (SOCP) formulation, which allows to relax the original non-convex power flow equations with a good compromise as it takes into account power losses while keeping a good computational tractability.

Each member n of the community is associated to a node of the network. This node is connected to a set of adjacent nodes denoted by $\mathcal{Y}_n$. Let's introduce the variables $\varphi_{n,j} = |i_{n,j}|^2$, the squared branch current flowing between nodes n and j, and $w_n = |v_n|^2$, the squared voltage magnitude at node n, where $i_{n,j}$ and v_n are complex substitutes. The time superscripts t have been omitted for the sake of clarity. Overall, the SOC equations of power flows for each time step t are:

$$
\begin{aligned}
&p_n = \sum_{j\in\mathcal{Y}_n} p_{n,j}, && \forall n \in \mathcal{N} && \text{(6.14a)}\\
&q_n = \sum_{j\in\mathcal{Y}_n} q_{n,j}, && \forall n \in \mathcal{N} && \text{(6.14b)}\\
&p_{n,j}^2 + q_{n,j}^2 \leq w_n \varphi_{n,j}, && \forall n, j \in \mathcal{Y}_n && \text{(6.14c)}\\
&p_{n,j} + p_{j,n} = R_{n,j}\varphi_{n,j}, && \forall n, j \in \mathcal{Y}_n && \text{(6.14d)}\\
&q_{n,j} + q_{j,n} = X_{n,j}\varphi_{n,j}, && \forall n, j \in \mathcal{Y}_n && \text{(6.14e)}\\
&w_j = w_n - 2\left(R_{n,j}p_{n,j} + X_{n,j}q_{n,j}\right) && && \\
&\qquad + \left(R_{n,j}^2 + X_{n,j}^2\right)\varphi_{n,j}, && \forall n, j \in \mathcal{Y}_n, && \text{(6.14f)}
\end{aligned}
$$

where p_n and q_n are the active and reactive power injected at node n, $p_{n,j}$ and $q_{n,j}$ are the active and reactive branch power flowing from n to j, while $R_{n,j}$ and $X_{n,j}$ are respectively the resistance and reactance of the branch connecting n and j.

The equality constraint from the original Optimal Power Flow (OPF) is relaxed into an inequality constraint (6.14c), hence including the cone interior space and ensuring convexity. Both terms of the equation should tend to be equal as the objective function tends to minimize power losses (cf. Section 6.3). The link between the prosumer model and the electrical network is described by the following equation:

$$
p_n^t = d_n^t + \sum_{a\in\mathcal{A}_n} x_{n,a}^t + s_{n,\mathrm{ind}}^t + s_{n,\mathrm{host}}^t. \tag{6.15}
$$

In contrast with (6.5), the variables v_n^t and $s_{n,\mathrm{mut}}^t$ coming from the mutualization of energy surpluses are not included as they are not physically linked to the prosumer n.

In turn, if the prosumer n has available storage space, the energy from mutualized excess storage can potentially be hosted on its device, which is captured by the variable $s^t_{n,\text{host}}$. The load p^t_n in (6.15) is referred as the physical net load on Figure 6.1.

6.2.3. Cost structure

The day-ahead power exchange scheduling problem is based on the minimization of the electricity costs, which consist in three contributions in the objective function. The first two are:

- The *commodity costs.* $(C^t_{\text{supp},i}(l^t_n))$
- The *upstream grid costs.* $(C^t_{\text{grid}}(L^t))$

They were already introduced in chapters 4 and 5, and detailed in 4.2.4. In this chapter, the grid costs are distinguished. The upstream component is unchanged, but it is complemented with:

- The *local grid costs:* they are directly based on the power exchanges inside the community. We assume that each member of the community is connected to the local distribution network (cf. Figure 6.1). Two different cost items are distinguished. One is based on the power losses in the lines while the second one is based on the line power flows in order to account for the wear and tear of the assets. The cost associated to each branch, formed by a pair of nodes $(j,k) \in \mathcal{B}$, where $\mathcal{B}$ is the set of all branches of the low-voltage entity, is expressed by:

$$\begin{aligned} C^t_{\text{loc}}(p^t_{j,k}) =& \Delta t \left(\gamma_{\text{loss}} \left(p^t_{j,k} + p^t_{k,j}\right)\right. \\ &\left. + \gamma_{\text{flow}} \cdot \delta_{j,k} \cdot |p^t_{j,k}|\right), \end{aligned} \tag{6.16}$$

  where γ_{loss} and γ_{flow} are respectively the costs of losses (in €/kWh) and line power flows (in €/kWh.km), while $\delta_{j,k}$ is the distance between the two nodes j and k.

Hence, the costs assumed in this chapter better reflect the true cost by accounting for locally incurred grid costs.

Overall, cooperating at the community level allows influencing positively all three costs contributions, i.e., through a joint consideration of (i) nodal energy exchanges (for commodity costs) and (ii)-(iii) line power flows (for grid-related fees).

6.3. Power exchange scheduling

6.3.1. Optimization problem

The day-ahead optimization problem minimizes the total cost of electricity consumption by the community. The benefits over a passive consumption or an individual minimization are the following:

i) By jointly planning their consumption, end-users can find the cost-optimal trade-off between arbitrage in the dynamic pricing (i.e., shift the load when prices are low), the upstream grid costs (which are minimized by smoothing the total load over time) and the local grid costs.

ii) By sharing their excess storage, they provide more flexibility to other community members, hence giving more potential to the first consideration above. Moreover, sharing excess energy from non-dispatchable sources not only provides free energy to other members but also decreases the grid costs by reducing the imports.

iii) By properly coordinating storage systems and power exchanges, power flows inside the community are contained, hence decreasing the related costs.

In practice, these three benefits are deeply correlated.

The objective function consists thus in jointly minimizing the following total community costs:

$$f(\Lambda) = \sum_{t\in\mathcal{T}} \Delta\tau \left[\sum_{i\in\mathcal{S}} \gamma_{\mathrm{com},i}^{t} L_i^{t+} + \gamma_{\mathrm{grid}} \left(L^t \Delta\tau\right)^2 \right.$$
$$\left. + \sum_{(j,k)\in\mathcal{B}} \left(\gamma_{\mathrm{loss}} \left(p_{j,k}^t + p_{k,j}^t\right) + \gamma_{\mathrm{flow}} \left(p_{j,k}^{t+} + p_{j,k}^{t-}\right) \delta_{j,k} \right) \right], \tag{6.17}$$

where $\Lambda = \cup_{n\in\mathcal{N}} \Lambda_n$ is the set of all decision variables including: $\mathbf{x}_{n,a}$, $\mathbf{s}_{n,\mathrm{ind}}$, $\mathbf{s}_{n,\mathrm{mut}}$, $\mathbf{s}_{n,\mathrm{host}}$ and $\mathbf{v}_n$ for each member n. The absolute active power value in (6.17) corresponds to $p_{j,k}^{+} = \max(p_{j,k}, 0)$ and $p_{j,k}^{-} = \max(-p_{j,k}, 0)$ (these constraints are not required in the algorithm because they are implicit).

The day-ahead energy exchange scheduling problem is therefore solved by:

$$\begin{aligned} &\min_{\Lambda} f(\Lambda) \text{ as in (6.17)} \\ &\text{s.t. (4.9)–(4.14), (6.5)–(6.13),} \end{aligned} \tag{6.18}$$

which is a convex optimization problem as both the constraints and the objective function are convex. It can be solved by standard algorithms such as interior-point or subgradient methods [76]. However, it can also be conveniently solved using a distributed algorithm (e.g., ADMM), thereby avoiding to resort on a third-party to guarantee the safe operation of the proposed design.

6.3.2. Cost distribution

Computing the minimum total cost of the community may not provide sufficient leverage for committing prosumers (cf. Chapter 5). Their collaboration should be incentivized by an adequate cost distribution (ensuring that all individuals have a financial interest in helping the community). In this chapter, two different options to distribute the cost savings arising from the global optimization (6.18) among community members are investigated.

It should be noted that introducing network fees in the definition of the cost distribution is inherently unfair as the contribution of individuals strongly depends on their geographical location and the grid infrastructure. Instead, we choose to distribute costs based on the commodity item, accounting for the consumption and sharing behaviors of the individuals. The network costs are then passed on proportionally to their net load in the second distribution scheme.

Individual optimum proportional billing

The first possibility consists in defining a cost distribution leading to a natural consensus. To that end, we exploit the fact that is possible to define a Nash equilibrium problem (cf. 3.3.2) while keeping the global optimum solution such as in Chapter 4. It has the advantage of not requiring a central authority nor any supervision. Hence, based on 4.3, the total cost is distributed according to a daily proportional billing. However, instead of adopting a net load proportional billing such as in 5.3.1, the distribution key accounts for the optimum commodity costs they would selfishly obtain without cooperation:

$$b_n = \frac{C_{\text{supp},i}^{\{n\}\star}}{\sum_{m\in\mathcal{N}} C_{\text{supp},i}^{\{m\}\star}} C_{\text{tot}}^{\{\mathcal{N}\}\star}, \tag{6.19}$$

where:

- $C_{\text{supp},i}^{\{n\}\star}$ is the commodity cost of prosumer n, customer of supplier i, when it optimizes its load selfishly (without cooperation).
- $C_{\text{tot}}^{\{\mathcal{N}\}\star}$ is the solution of the optimization problem (6.18).

Note that the superscripts specify the coalition involved and if the optimal value is considered (*) or not. Besides, all the costs considered for billing derive from the computations across all time steps.

Compared to the net load proportional billing (5.10), the individual optimum proportional billing (6.19) individualizes the resulting prices by accounting for the choice of the supplier. Therefore, the appropriate supplier choice lies in the individual instead of the community.

The billing problem (6.19) can also be expressed as the solution of the following Nash equilibrium problem.

- *Players*: all N community members
- *Strategies*: all the possible strategies of the decision variable set Λ_n
- *Payoffs*:

$$P(\Lambda_n; \Lambda_{-n}) = -b_n = -\frac{C_{\text{supp},i}^{\{n\}\star}}{\sum_{m\in\mathcal{N}} C_{\text{supp},i}^{\{m\}\star}} f(\Lambda), \tag{6.20}$$

with $\Lambda_{-n} \triangleq [\Lambda_1, \ldots, \Lambda_{n-1}, \Lambda_{n+1}, \ldots, \Lambda_N] = \Lambda \setminus \{\Lambda_n\}$, the vector containing all the decision variables except those of n.

Given the strict convexity of (6.20), the Nash equilibrium exists and is unique, cf 4.3. Consequently, the solution can be obtained by solving (6.19), which requires the computation of $N+1$ problems: N selfish problems $C_{\text{supp},i}^{\{n\}\star}$ and the main cooperative optimization $C_{\text{tot}}^{\{\mathcal{N}\}\star}$.

Overall, the Nash equilibrium reached by the individual optimum proportional billing cannot reward participants according to their contribution to the savings. The solution tends to be more egalitarian (i.e., the costs are equally distributed among clients, although accounting for the supplier choice) and could thus be considered less incentivizing (since the offered flexibility is not properly valued at the individual level, but shared among all members). To alleviate this effect, the cost distribution resulting from a derivative of the Shapley value is also studied.

Shapley value and marginal cost pricing

The Shapley value is a concept of cooperative game theory assigning a share of the gains obtained by forming coalition of different actors [132], cf. 3.3.4. It allows to remunerate end-users with payoffs that correspond to their individual contributions to the gains of the community, and is characterized by a collection of desirable properties introducing a certain degree of fairness.

Remark 6.1. It is because the Shapley value has the reputation to be fair that a cost distribution has been based on it. However, it is not used in its common coalitional game setting. Instead, it aims at defining a marginal cost pricing in a non-cooperative setting. Hence, it is very similar to the VCG-based marginal cost pricing of Chapter 5, although the latter is much less demanding to compute. Besides, the contributions presented in Chapter 5 came after these developments and it opted for simplifying the cost distribution method while keeping the same philosophy.

Let $\mathcal{Q} \subseteq \mathcal{N}$, define a coalition of size $|\mathcal{Q}|$ and the characteristic function $v(\mathcal{S}) : 2^{\mathcal{N}} \to \mathbb{R}$ describe the savings associated to the formation of $\mathcal{Q}$. A cooperation game is defined by the pair $(\mathcal{N}, v)$ and the Shapley value ϕ_n attributed to a given player n is given by

$$\phi_n(v) = \sum_{\mathcal{Q} \subseteq \mathcal{N} \setminus \{n\}} \frac{|\mathcal{Q}|!(N - |\mathcal{Q}| - 1)!}{N!} [v(\mathcal{Q} \cup \{n\}) - v(\mathcal{Q})]. \tag{6.21}$$

Although all the 2^N combinations of coalition are necessary to compute the Shapley values, it should be noted that the only coalition possible in the cost structure considered in this chapter is the grand coalition, that is $\mathcal{N}$. Indeed, as previously mentioned, the network costs cannot be directly echoed to an individual or a subset of the community as the power flows depend on the joint load of all clients. Based on all the previously mentioned considerations, a billing with a double distribution key is proposed. On one hand, the commodity costs are distributed according to their Shapley value (numerator of the first term in (6.22)) obtained on the optimum commodity cost and scaled to the actual commodity costs (remaining of the first term). On the other hand, the grid costs are allocated proportionally to the net load. The total costs b_n incurred by participant n under the Shapley-based pricing are thus:

$$b_n = \frac{C^{\{n\}\star}_{\text{supp},i} - \phi_n(v)}{C^{\{\mathcal{N}\}\star}_{\text{supp}}} C^{\{\mathcal{N}\}}_{\text{supp}} + \frac{l_n}{\sum_{m \in \mathcal{N}} l_m} C^{\{\mathcal{N}\}}_{\text{grid}}, \tag{6.22}$$

where :

- $C^{\{\mathcal{N}\}}_{\text{grid}}$ is the sum of $C^{\{\mathcal{N}\}}_{\text{up}}$ and $C^{\{\mathcal{N}\}}_{\text{loc}}$, the total upstream and local grid costs.
- $C^{\{\mathcal{N}\}\star}_{\text{supp}}$ is the optimum community commodity cost obtained by solving a truncated version of (6.18) which considers commodity costs only (i.e., excluding network costs).

- $C_{\text{supp}}^{\{\mathcal{N}\}}$ and $C_{\text{grid}}^{\{\mathcal{N}\}}$ are respectively the actual commodity and grid costs of the community. They are obtained by extracting respectively the commodity and grid costs after solving the full version of (6.18).

and with $C_{\text{supp}}^{\{\mathcal{N}\}} + C_{\text{grid}}^{\{\mathcal{N}\}} = C_{\text{tot}}^{\{\mathcal{N}\}\star}$, the solution of the optimization problem (6.18). Note that the computation of the 2^N combinations of coalition costs is sufficient to calculate the bills.

Benchmark

In order to quantify the added value of both cost distributions, i.e., (i) the individual optimum proportional billing, and (ii) the Shapley-based payoffs, a proper benchmark that quantifies what can be achieved using individualized optimal policies (without collaboration) is introduced. To that end, a framework (iii) is considered where each end-user individually minimizes its cost objective function. Practically, two different individual strategies are analyzed (in Section 6.4), which respectively account for the minimization of the commodity costs and the minimization of the grid costs over the daily scheduling horizon. Table 6.1 summarizes the distribution keys of the three methodologies.

Table 6.1.: Distribution key summary.

	Optimum (i)	Shapley (ii)	Indiv. strat. (iii)
$C_{\text{supp}}^{\{\mathcal{N}\}}$	$\dfrac{C_{\text{supp},i}^{\{n\}\star}}{\sum_{m\in\mathcal{N}} C_{\text{supp},i}^{\{m\}\star}}$	$\dfrac{C_{\text{supp},i}^{\{n\}\star} - \phi_n(v)}{C_{\text{supp}}^{\{\mathcal{N}\}\star}}$	$\dfrac{C_{\text{supp},i}^{\{n\}\star}}{\sum_{m\in\mathcal{N}} C_{\text{supp},i}^{\{m\}\star}}$
$C_{\text{grid}}^{\{\mathcal{N}\}}$		$\dfrac{l_n}{\sum_{n\in\mathcal{N}} l_n}$	$\dfrac{l_n}{\sum_{n\in\mathcal{N}} l_n}$

Regarding both cooperation schemes (i)-(ii), it is necessary that participants pledge their contribution and be transparent about their engagement for the predefined consensus. This consideration could, for instance, be treated by the use of a blockchain.

6.4. Case studies

The proposed framework is analyzed on a benchmark composed of $N = 3$ prosumer nodes. Each of them is assigned to a load representing the equivalent of about five low-voltage end-users, hence summing up to 15 end-users, to reflect the conditions that could be found on a residential feeder (typical Belgian feeder such as in [133]) and obtain representative power flows in the low-voltage network. The feeder is assumed to be one kilometer long and is fully radial. Table 6.2 summarizes the main features of the nodal loads, i.e. details about their electrical equipment, their choice of supplier (1: cheaper nights, 2: cheaper days), and their potential in terms of load flexibility. Note that in order to ease the observations, the scheduling takes place for a day comprising four time slots. Additional information regarding the LV network, the load constraints, and the tariffs applied can be found in the online appendix [134].

Note that all scenarios were implemented with Julia. Optimizations were achieved with the Mosek™ solver [135] and the solutions were obtained within 5s.

Table 6.2.: Summary of the end-users characteristics.

	PV	ESS	Supplier	Shiftability
Node 1	X	X	1	28%
Node 2	X		1	42%
Node 3		X	2	56%

In what follows, in 6.4.1, the total costs (aggregated at the community level) between individual and cooperative approaches are compared. Secondly, 6.4.2 analyzes in more detail how the proposed cooperative market design allows to cost-efficiently share the available flexibility among end-users, thus highlighting the interest of differentiating physical from virtual energy exchanges. Then, based on the scheduling solution obtained by the global optimization, 6.4.3 investigates how the costs are distributed among individuals (using both the individual optimum proportional billing and the Shapley-based billing). Finally, the benefits of cooperation are highlighted in 6.4.4 by comparing the global cooperative optimization with a traditional framework where each end-user optimizes its own electricity bill

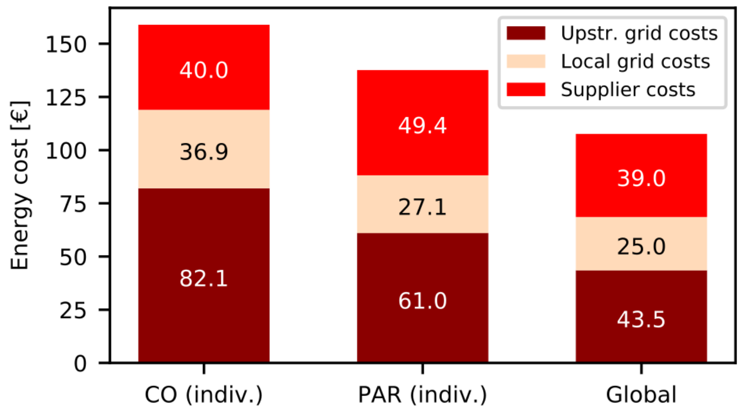


Figure 6.3.: Comparison of the total cost division for different load management scenarios.

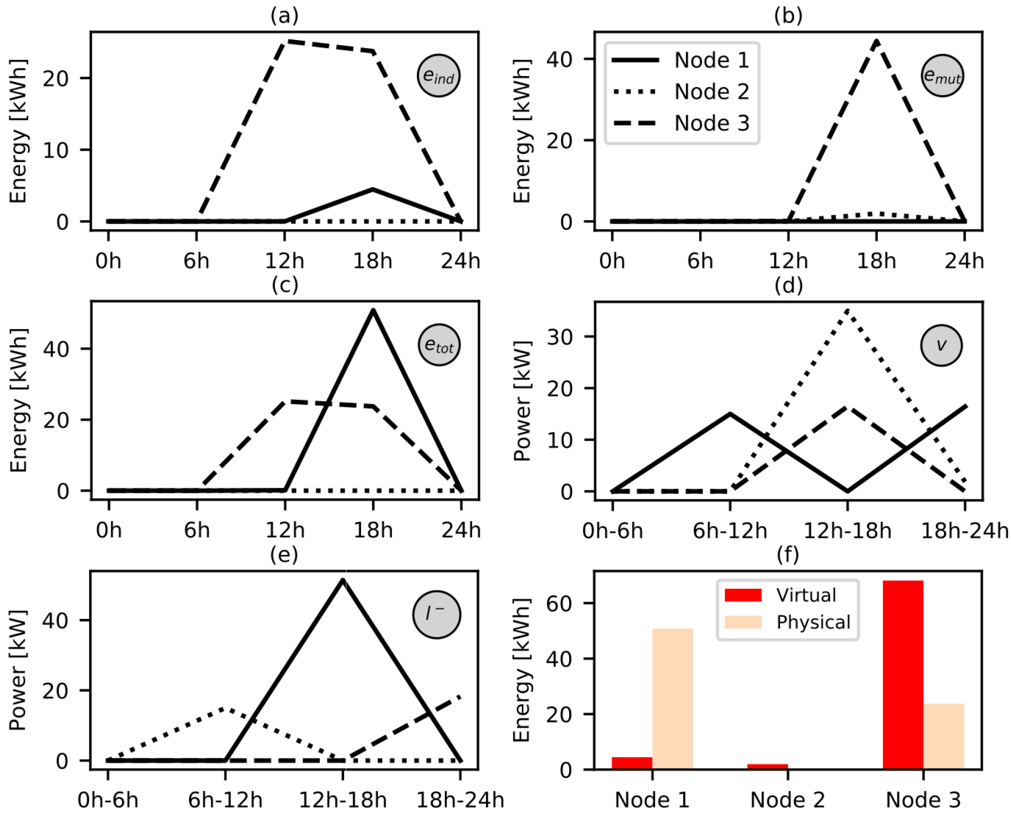


Figure 6.4.: Temporal evolution of the personal available energy stored locally $\mathbf{e}_{\mathbf{n},\text{ind}}$ (a), the personal available energy stored remotely $\mathbf{e}_{\mathbf{n},\text{mut}}$ (b), the total energy stored locally $\mathbf{e}_{\mathbf{n},\text{tot}}$ (c), the allocated portion of excess power $\mathbf{v}_{\mathbf{n}}$ (d) and the virtual negative net load $\mathbf{l}^-_{\mathbf{n}}$ (e) $\forall n$. Virtual (available for personal use) and physical ESS state of charge at 18h (f).

with fixed grid fees.

6.4.1. Day-ahead scheduling of end-users

A comparison of the total costs for three different load management approaches is displayed on Figure 6.3. The first two are the individual strategies briefly presented in Section 6.3.2 as a benchmark to quantify the added value of the global optimization. Firstly, a basic situation is considered, referred to as Commodity-Only (CO), where each individual optimizes its own commodity costs (by using its own flexibility to shift the load during off-peak prices), thus obtaining $C^{\{n\}\star}_{\text{supp},i} \forall n$. The second approach applies the peak-to-average ratio, where end-users try to integrate network costs in their own objective function. However, since these two policies do not involve cooperation at the community level, there is a lack of global vision that limits their potential. This effect is quantified by solving the global optimization (6.18), which yields the total commodity costs of the community $C_{\text{supp}}^{\{\mathcal{N}\}}$.

Interestingly, introducing the peak-to-average ratio (PAR) allows decreasing the total costs in comparison with the individual minimization of commodity costs (Figure 6.3). Indeed, by exclusively focusing on commodity costs, end-users tend to concentrate their energy consumption at specific time slots (when dynamic prices are the lowest). This behavior results in consumption peaks that significantly increase the grid costs. This effect is further exacerbated when all clients rely on the same incentives (i.e., when they have similar commodity tariff schemes). The PAR optimization alleviates this issue through a better homogenization of the energy exchanges along the day, hence leading to higher commodity costs but to a significant reduction of both grid fees components (within and outside the community).

Overall, it is important to emphasize that these two individual approaches probably represent optimistic individual behaviors as they assume that each individual will optimize its flexible resources on a daily basis. A more passive behavior of end-users or a lack of control means would surely yield higher costs. These optimistic outcomes from both individual optimizations are, nonetheless, outperformed by the global optimization (6.18) since the latter takes advantage of the flexibility of all available resources. The cooperative framework achieves minimal costs for the three contributions of the final electricity bill. This arises from the inclusion of energy sharing mechanisms that enable sharing optimally inter-participant flexibility, i.e., excess mutualized storage space and photovoltaic generation. As the billing of each individual is based on the total cost, decreasing net imports of other individuals through the provided flexibility benefits the whole community.

6.4.2. Benefits of the community-based optimization

When focusing on the global optimization solution, the relevance of the different energy-sharing mechanisms can be easily highlighted. For instance, node 1, at which end-users present a poor load shiftability (cf. Table 6.2), could not leverage its DERs (excess production and storage), available mostly in the afternoon, without sharing. Indeed, most of its load must be planned in the night and morning. In contrast, it can be observed that when the global optimization is considered, node 1 hosts a significant amount of energy for node 3 and a small quantity for node 2. Indeed, from (a-c) on Figure 6.4, it can be identified that most of the energy stored at node 1 is not for personal use ($e_{1,\text{tot}}^{18h} \gg e_{1,\text{ind}}^{18h}$) and that $e_{2,\text{mut}}^{18h}$ and $e_{3,\text{mut}}^{18h}$ are both positive. Furthermore, nodes 2 and 3 benefit from excess energy originating from the PV generation of node 1 (cf. (d-e) on Figure 6.4). However, beyond benefiting indirectly from the underlying decrease of the grid costs, node 1 also gets excess energy from nodes 2 and 3 in the morning and evening, respectively. Note that no energy is stored at night as a significant amount of energy must be supplied and transit from the upstream grid. An additional storage load would yield a considerable increase in the associated bill.

Furthermore, from Figure 6.4(f), the distinction between physically and virtually-stored energy can be highlighted. The difference between both values corresponds to either the hosted energy $e^t_{n,\text{host}}$ by an individual if there is an excess of physical storage space or the remote energy of an individual $e^t_{n,\text{mut}}$ otherwise.

6.4.3. Effects of different cost distributions

After the minimization of the total costs (6.18), it is necessary to fairly and optimally distribute these costs to all members of the community. Both cost distribution keys (presented in 6.3.2) are compared, and outcomes are illustrated in Figure 6.5 for the final electricity bill of the $N = 3$ nodes of the network. For the sake of comparison, results from the CO and PAR optimizations are also depicted.

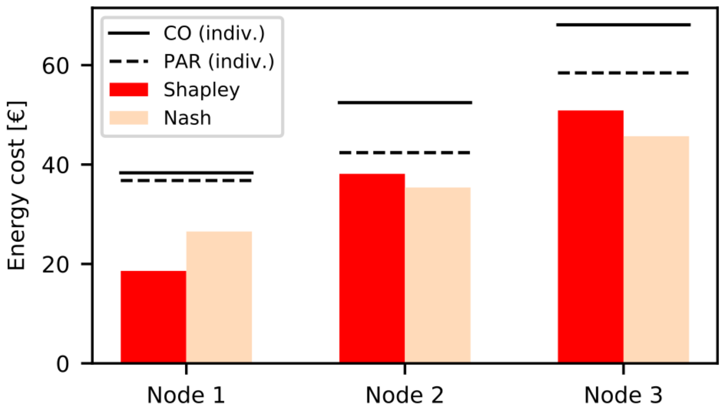


Figure 6.5.: Comparison of the final electricity bills of end-users with the different cost distribution keys.

It can be observed that the gains arising from the PAR optimization lead to costs reduction for all three nodes of the community with respect to the sole minimization of commodity costs. All consumers have thus a financial incentive for incorporating grid fees into their day-ahead scheduling. However, these gains are much lower for node 1, which can be explained by the fact that both CO and PAR optimizations result in a similar planning solution (they schedule most of the flexible load when the PV injection is high).

Overall, although improving the way end-users allocate their resources can be profitable, these are incentivized into forming local communities to further reduce their electricity bill.

When global optimization is considered, the two cost distributions yield different trends. Under the present scenario, the resulting gains under the individual optimum proportional solution (6.19) generate smaller costs differences between community members than the outcome obtained with the Shapley-based billing. Indeed, the latter option tends to reward actors who contribute most to the decrease of the commodity, whereas the former solution values the potential flexibility. In that regard, the significant contribution of node 1 highlighted in 6.4.2 is better valued with the Shapley-based payoffs. The two options thereby give different incentives to fully leverage the flexibility of low-voltage prosumers.

Table 6.3.: Impact of storage on costs

Energy cost [€]	Node 1	Node 2	Node 3	Total
ESS / Nash	24.25	35.97	47.42	107.64
ESS / Shapley	19.75	36.39	51.50	
No ESS / Nash	26.7	33.70	49.07	109.47
No ESS / Shapley	21.38	36.49	51.60	

These observations can be further highlighted by modifying the available storage capacity. Table 6.3 gives the impact of storage on the energy cost for each node. From this table, we observe that if there is no available storage for nodes 1 and 3, the total electricity cost of the community is increased. In particular, the solution given by the individual optimum proportional billing raises the electricity bill of both clients (1 and 3), while the Shapley-based outcome only affects node 1. Indeed, node 1 has less load flexibility than other members and is thereby more impacted by the Shapley methodology that rewards the actual mobilized flexibility. It can be also noted that the absolute amount of the bill with zero ESS capacity could be much higher during a day with higher PV production. This results from the increased grid costs related to the higher injection.

6.4.4. Benefits of sharing the grid fees at the community level

In this section, the outcomes obtained with our proposed cooperative approach (which properly considers non-linear grid costs) are compared with a benchmark representative of the current reality in most places worldwide. Hence, the latter design assumes an individual approach where each end-user optimizes its own cost function, in which grid costs are linearly integrated (proportional to the energy costs).

Table 6.4 shows, for the same initial benchmark conditions, the relative differences of billings for various fixed grid rates (price per unit of consumed energy) in comparison to our cooperative approach with aggregative non-linear grid costs (a positive value corresponds to an increase of the costs).

Table 6.4.: Costs difference between aggregative and linear grid fees.

% costs difference	Node 1	Node 2	Node 3
0.208 €/kWh	14.3	6.5	5.1
0.30 €/kWh	48.5	37.2	35.2
0.10 €/kWh	-25.8	-29.5	-30.2

Firstly, it can be observed that even the optimistic case of a fixed grid price corresponding to the mean price obtained under the global optimization (0.208 €/kWh) leads to an increase of the billing for the three considered nodes. This comes from the fact that individual optimization leads to higher imports (no sharing of surpluses). In reality, as there is no optimization across grid costs, the incurred costs should be even higher (cf. individual approaches on Figure 6.3), and a higher fixed rate could be expected. In that regard, the results show that increasing the fixed grid price (0.30 €/kWh) has a greater impact on node 1. This arises from its lower flexibility potential that prevents it from adapting its schedule. On the other hand, lower grid fees, i.e., 0.10 €/kWh (which could reflect a more efficient planning and operation of the whole system), are smoothing the differences among nodes.

So far, the results of an aggregative non-linear grid costs framework have been exposed considering both the individual and community approaches (cf. Sections 6.4.1 to 6.4.3), while this section introduced a linear grid costs framework under the individual approach. It should be noted that a community approach using linear grid costs could have been considered, but it is not straightforward and implies structural changes.

Indeed, the aggregative grid costs framework formalizes the interdependence of individuals sharing the same asset: the grid infrastructure. This naturally promotes cooperation since all the individual problems are linked to form a global problem. In this way, the mutualizing mechanisms are driven by the optimization of the global problem. On the contrary, linear grid costs leave the individual problems separable. Introducing mutualization mechanisms in such a framework is not driven by a natural consensus. On one hand, the PV excesses, although they can be possibly shared if the actors are not penalized (e.g., no injection grid costs if the surplus is shared), find no natural consensus on a distribution key. On the other hand, an individual finds no interest in sharing its surplus of storage space unless it gets a financial advantage. Another market model should therefore be developed in such a case. This further highlights the benefits of this approach that reflects the true costs. Indeed, it properly considers the aggregative nature of grid fees and the zero-marginal cost of sharing energy surpluses.

6.5. Insight on other billing schemes

Continuous proportional billing (cf. 5.3.1) was applied to the framework of this chapter. Similarly to its formulation in the context of Chapter 5, the Nash equilibrium problem does not lead to computing the global (social) optimum. In addition, shared constraints are introduced into the problem. They correspond to constraints spanning across variables of multiple members. Hence, equations (6.10), (6.12) and (6.15), which govern the sharing of excess storage space, excess photovoltaic production, and the microgrid power flows, turn the game into a *generalized Nash equilibrium problem*, cf. 3.3.2.

Preliminary solutions are consistent with the observations of Chapter 5. There is very little inefficiency in comparison to the global minimum. Hence, the same conclusions can be transposed to this framework. However, more advanced mathematical developments need to be consolidated to ensure the convergence of the algorithm. They are based on the reduction of the problem into a suitable VI problem that allows deriving variational solutions, cf. 3.3.3. Even so, it appears to be complicated to generalize mathematical proofs when going beyond simplistic scenarios. There has been little recent literature on the subject. Advances on QVI or alternative computation methods could possibly be considered. In the present case, convergence was obtained by empirically tuning the algorithm parameters.

It should be noted that any other billing scheme that does not apply a distribution key (proportion) to the total community cost leads to the same considerations.

6.6. Conclusions

This chapter has presented an advanced framework optimally leveraging the flexible resources distributed throughout low-voltage (residential) networks. The solution aims at alleviating issues from centralized and fully distributed paradigms by relying on local communities (using the existing infrastructure) and building on the mutualization of excess resources. The chapter firstly introduced a global optimization framework, substantially similar to the energy consumption scheme developed in chapters 4 and 5, but complemented with community-related considerations and a finer reflection of local network costs. Then, two different techniques, based on the individual optimum and the Shapley value respectively, that distribute the total gains among end-users were developed.

Results highlight the significant cost savings of forming a community with respect to the individual optimization of assets. Then, we observe that distributing the resulting costs using the Shapley value allows to endogenously reward the actors that contribute most to the costs reduction through their mobilized flexibility, whereas the individual optimum proportional billing tends to reward the potential flexibility consent. The two implementations thus offer different incentives to fully leverage the flexibility of the low-voltage prosumers.

The framework presented in this chapter corresponds to the definition of responsible energy community as defined in Box 2.6. There are, however, notable prospects that may enrich the concept. Firstly, it seems indispensable to address the other time frames that communities face along with their respective issues. The next chapter provides an insight into the investment horizon and the intraday resolution. Secondly, adapting scenarios with shared constraints to multi-temporal billings with a flexible, efficient, and more systematical computational framework may provide a significant boost to such implementations.

This chapter has led to the following publication:

M. Hupez, J-F. Toubeau, Z. De Grève and F. Vallée, "A New Cooperative Framework for a Fair and Cost-Optimal Allocation of Resources Within a Low Voltage Electricity Community," in *IEEE Transactions on Smart Grid*, vol. 12, no. 3, pp. 2201-2211, 2021.

CHAPTER 7.

From long-term planning to settlement

In the previous chapters, the same time horizon has been addressed: the day-ahead scheduling. Although it has laid a solid foundation for an alternative energy exchange scheme within a community, the day-ahead scheduling scenarios alone are insufficient to maintain a long-lasting coherent entity.

A holistic solution should address longer-term and shorter-term horizons. In particular, investment in DERs and intraday energy exchange settlement are the most relevant. The latter can secure the developed framework by preventing multi-horizon strategy, and the former provides additional benefits related to the optimized use of resources. Both can thus increase prosumer engagement and the stability of the communities.

Addressing fully these two topics requires developments that go beyond this thesis. However, this chapter provides some concrete ideas and elements of formulation that strengthen the core proposition.

Hence, this chapter aims at sketching scenarios addressing long-term planning and settlement within a responsible energy community. The first section highlights the needs and benefits in more detail. Section 7.2 develops different alternatives when it comes to planning in a coordinated manner the investment in DERs. Then, Section 7.3 examines the intraday horizon and gives an insight on the possible options for settlement. Finally, Section 7.4 provides an overview of the energy exchange scheduling on the multiple time horizons.

7.1. Introduction

Naturally, the responsible energy communities framework, such as exposed in the previous chapter, promotes more sensible investments. Indeed, investing in DERs for personal profit is even more beneficial because these assets can be mobilized for other community members when they are not used by their owners. The mutualization generates an overall decrease of the grid costs, which is shared among all members and therefore favors the asset owner as well.

However, it is not straightforward for an individual to invest while taking into account this last consideration. The benefits generated on the assets' lifetime, which should be compared to the investment cost, are not easily quantifiable. Especially those that are indirect, i.e., due to the decrease of the grid expenses and the sharing by other community members, cannot be determined individually. Prosumers could, for instance, over-invest if they underestimate the free resources they can get from other members and vice versa. Hence, it is judicious to think of the investment problem as a concerted exercise between community members.

Once again, strategy is involved in such an exercise. In the end, end-users wish to minimize their bill (or at least, it is the objective that will convince as many end-users as possible), but it is dependent on the others' solution. Such an interdependent multi-agent optimization can be formulated as a game.

The investment problem must take into account the daily billing rules. Hence, the energy exchange scheduling scenario, as developed in Chapter 6, is involved. Instead of defining a day-ahead solution, it is incorporated as a subroutine providing the expected aggregated daily expenses.

Besides, using the energy exchange scheduling scenario as an operational tool for the daily planning of a community raises the crucial question of how the deviations should be handled. Inevitably, unplanned events may occur, and the expected individual DER production can diverge significantly (e.g., day-ahead forecasts are never fully accurate). Depending on the resolution rules and the subsequent billing adaptations, it may also influence the day-ahead scheduling.

7.2. Coordinated investment planning in responsible energy communities

7.2.1. Investment within energy communities

To this day, most implementations of energy communities do not tackle the investment problem. Usually, a set of prosumers and possibly other actors (local producers) holding preexisting assets join to form a collective, e.g., [136]. The regulatory framework is locally adapted to account for the local dimension of such associations and may exonerate part of the taxes and/or grid fees on the local energy exchanges. Besides, the majority of these communities do not integrate any active load management scenarios. The local autoconsumption is passively incentivized (no DR algorithm) because of the lower prices on local energy procurement.

There are, however, examples of projects that aim at developing energy systems with local community ownership, cf. [137], [138]. They feature many different organizational arrangements, project purposes, and technologies. Unfortunately, such diversity does not offer a consistent framework and often overlooks the potential effect of load management on the needs.

A more consistent scheme that is developing is the community-based virtual power plant. Such a collective usually integrates an energy management system acting as an aggregator, manager, and supplier of the locally produced energy (e.g., cVPP [139]). The denominator of such collectives is aggregation and is meant to fit the market design. It can be seen as a local version of the FPP described in 2.5.2. The investment problem is thus directly linked to the business plan of the virtual power plant.

In this section, different investment planning configurations are developed. They are directly integrated into the model of responsible energy communities, hence complying with its main hypotheses and guidelines (cf. 4.2.1). In addition, they directly account for the operational load management scenarios of the proposition, namely the energy exchange scheduling. Designing such investment models gives an added value to responsible energy communities as it reinforces its objectives. The proposition of this section is, however, a first draft and does not mean to answer all the challenges that such a task involves. This subject deserves much more attention and future developments.

7.2.2. Planning framework

Overall structure

The current literature extensively addresses planning problems, e.g., energy system optimization models [140] and generation expansion planning [141]. These approaches attempt to define an investment plan for aggregations of consumers and producers.

Here, the proposition of the planning problem is the following:

The planning aims at defining the adequate investment in DERs that is expected to minimize the total electricity supply costs over the assets' lifetime. The proposed framework is articulated around two modeling blocks, cf. Figure 7.1.

i **Initial investment model.** It corresponds to variables and parameters related only to the capital expenditures (CAPEX). They address the initial time of investment.

ii **Daily costs estimation model.** It relies on the day-ahead energy exchange scheduling mechanism. It is used for computing solutions employing typical days scenarios (cf. 7.2.3) covering the life expectancy of the related assets. It provides thus an estimate of the operational expenditures (OPEX).

The outcomes of the planning optimization process are the DER capacities that should be installed to minimize the overall expected electricity supply cost. Operational variables remain endogenous. They are only computed to reproduce representative statistics of scheduling scenarios.
The main novelty and contribution of this chapter is to embed the billing problem in the investment problem. This is highly valuable as the billing problem will affect the decisions of consumers regarding their investments.

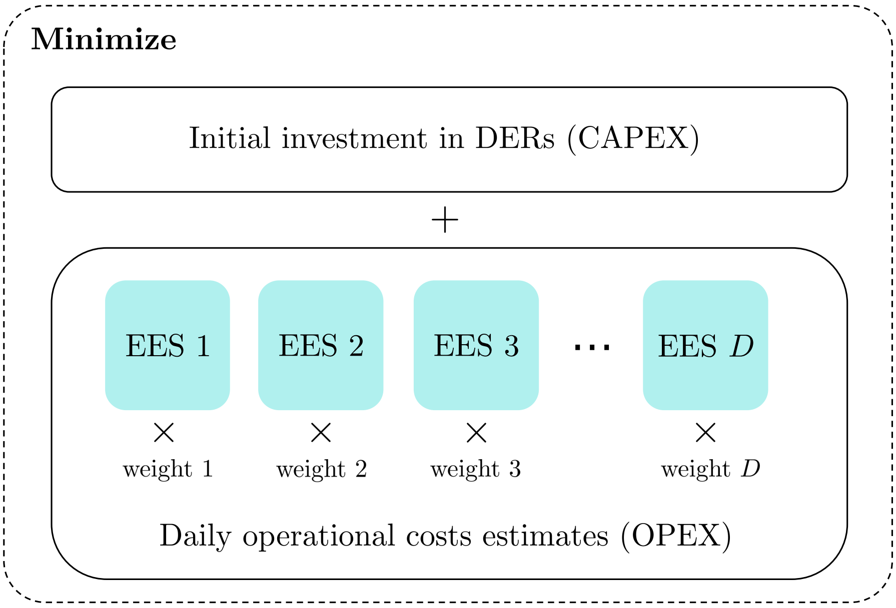


Figure 7.1.: Planning tool structure. Energy exchange scheduling is used to produce OPEX estimates using weighted typical days.

Investment strategies

Considering the circumscription to residential entities, which is adopted in the present work, and accounting for the developing trends in DERs, two types of assets are considered.

- *Photovoltaic panels.* They can be rooftop installations or small photovoltaic farms.

- *Energy storage systems.* They can be from one individual unit to larger storage facilities.

Hence, the investment problem aims at defining an optimized quantity for both assets. Two different investment philosophies are distinguished.

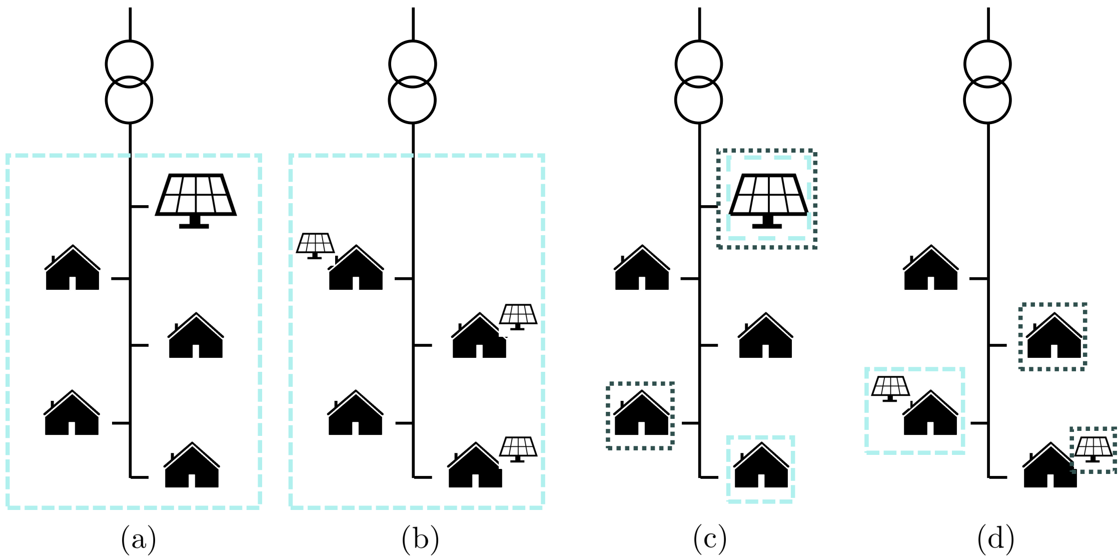


Figure 7.2.: Dashed boxes indicate ownership. (a) Joint investment and centralized installation. (b) Joint investment and decentralized installation. (c) Individual investments and centralized installation. (d) Individual investments and decentralized installation (possibly hosted by another individual).

i **Joint investment.** A shared ownership is considered. Community members agree on an a priori optimized investment. They adopt a more or less sophisticated cost distribution key. Then, adjustments are made according to the actual use of the resources (through the daily billing or periodical settlements).

ii **Individual investment.** Individuals define their optimized investment irrevocably (although accounting for the mutualization mechanisms). They have the ownership of the assets.

Physically, DER facilities can be installed in two forms.

i **Centralized installation.** DERs are either constrained to a certain location on the network or (an) optimized position(s). Note that the two investment philosophies described above are compatible with this form of installation as decribed in 7.2.3 and depicted on Figure 7.2(a)+(c). Indeed, an individual may not have the appropriate space to install its assets.

ii **Decentralized installation.** Either the joint investment leads to installations hosted by individuals, i.e., no centralized node (cf. Figure 7.2(b)), or each individual installs its DERs at home or agrees with another individual to host its facilities (cf. Figure 7.2(d)). In the latter case, an additional mechanism should probably be added to reward the host.

There are thus four different investment configurations, which will lead to different planning variables and formulations.

7.2.3. Extended model

All operational variables, state variables, and parameters (from the day-ahead energy scheduling scenarios) were already described in the previous chapters. They are reused to feed the investment optimization process. Instead of providing schedules, they provide daily cost estimates (cf. 7.2.2) of the operation of the defined community framework. In addition to the operational model, investment variables and parameters need to be defined.

Prosumer model adaptations

Two new decision variables complement the framework of the day-ahead energy exchange scheduling.

- *The non-dispatchable generation installed capacity (kWc).* It can be either a single variable ($J^{\mathrm{c}}_{\mathrm{gen}}$) if a centralized joint investment is considered, or N different variables in the three other cases (cf. Table 7.1). Variables $J^{\mathrm{d}}_{\mathrm{gen},n}$ denote the share of collective capacity that is installed at node $n \in \mathcal{N}$. When personal investment is considered, variables $I^{\mathrm{c}}_{\mathrm{gen},n}$ represent the share owned by member $n \in \mathcal{N}$ centrally located on a dedicated node, whereas $I^{\mathrm{d}}_{\mathrm{gen},n,m}$ describes the personal device of member $n \in \mathcal{N}$ physically located on node $m \in \mathcal{N}$.
- *The energy storage system capacity (kWh).* Similarly to the non-dispatchable generation installed capacity, it can be either a single variable ($J^{\mathrm{c}}_{\mathrm{st}}$) if a centralized joint investment is considered, or N different variables in the three other cases (cf. Table 7.2). Variables $J^{\mathrm{d}}_{\mathrm{st},n}$ denote the share of collective capacity that is installed at node $n \in \mathcal{N}$. When personal investment is considered, variables $I^{\mathrm{c}}_{\mathrm{st},n}$ represent the share owned by member $n \in \mathcal{N}$

Table 7.1.: Summary of installed capacity variables (non-dispatchable generation) depending on the investment framework.

Installed capacity	Centralized	Decentralized
Joint investment	$J^{\mathrm{c}}_{\mathrm{gen}}$	$J^{\mathrm{d}}_{\mathrm{gen},n}$
Individual investment	$I^{\mathrm{c}}_{\mathrm{gen},n}$	$I^{\mathrm{d}}_{\mathrm{gen},n,m}$

Table 7.2.: Summary of installed capacity variables (energy storage system) depending on the investment framework.

Installed capacity	Centralized	Decentralized
Joint investment	$J^{\mathrm{c}}_{\mathrm{st}}$	$J^{\mathrm{d}}_{\mathrm{st},n}$
Individual investment	$I^{\mathrm{c}}_{\mathrm{st},n}$	$I^{\mathrm{d}}_{\mathrm{st},n,m}$

centrally located on a dedicated node, whereas $I^{\mathrm{d}}_{\mathrm{st},n,m}$ describes the personal device of member $n \in \mathcal{N}$ physically located on node $m \in \mathcal{N}$.

It is easily transposable to broader frameworks, including, for instance, wind turbines. The methodology is unchanged, and the objectives are preserved as long as the asset is included in the network of the concerned entity.

Depending on the introduced investment parameters, some definitions need to be modified.

If individual investments are considered (centralized or not), the individual (virtual) net load may be modified to

$$l_n^t = \kappa_n^t - \sigma_{\mathrm{cast}}^t I_{\mathrm{gen},n} + x_n^t + s_{n,\mathrm{ind}}^t - v_n^t + s_{n,\mathrm{mut}}^t, \tag{7.1}$$

where σ_{cast}^t is a state variable characterizing the load factor of the photovoltaic system (directly dependent on the solar irradiation), and $I_{\mathrm{gen},n}$ is generic for centralized and decentralized installations of generation. The other baseload component, i.e., the non-flexible load thus appears explicitly (κ_n^t), cf. (4.5).

Storage constraints (6.6)–(6.7) need to be adapted as well. They include the installed capacity investment variables.

$$0 \preceq \mathbf{e}_{n,\mathrm{ind}} + \mathbf{e}_{n,\mathrm{host}} \preceq I_{\mathrm{st},n} \qquad \forall n \in \mathcal{N} \tag{7.2}$$

$$-\mu^{\mathrm{dis}} I_{\mathrm{st},n} \preceq \mathbf{s}_{n,\mathrm{ind}} + \mathbf{s}_{n,\mathrm{host}} \preceq \mu^{\mathrm{ch}} I_{\mathrm{st},n} \qquad \forall n \in \mathcal{N}, \tag{7.3}$$

where μ^{ch} and μ^{dis} are parameters characterizing the charging and discharging power rates, and $I_{\mathrm{st},n}$ is generic for centralized and decentralized installations of storage. Equations (6.8)–(6.11) complement the storage model for the day-ahead energy exchange scheduling.

If joint investment is considered, the (virtual) net load becomes

$$l_n^t = \kappa_n^t + \sum_{a \in \mathcal{A}_n} x_{n,a}^t - v_n^t. \tag{7.4}$$

Moreover, the available shared power (6.12) is adapted to

$$\sum_{n \in \mathcal{N}} v_n^t \leq \sum_{n \in \mathcal{N}} l_n^{t-} + \sigma_{\text{cast}}^t J_{\text{gen}} + s_{\text{joint}}^t, \qquad \forall t \in \mathcal{T}, \tag{7.5}$$

where s_{joint}^t denotes the collective battery storage variable, and

$$J_{\text{gen}} = \begin{cases} J_{\text{gen}}^{\text{c}} & \text{if physically centralized,} \\ \sum_{n \in \mathcal{N}} J_{\text{gen},n}^{\text{d}} & \text{if physically decentralized.} \end{cases} \tag{7.6}$$

Indeed, when joint planning is considered, collective production must be added. Although the photovoltaic facility can be physically centralized or not, it is virtually centralized (collectivized).

In addition, the physically or virtually centralized battery is governed by

$$0 \leq \sum_{t=1}^{\bar{t}} s_{\text{joint}}^t \Delta\tau + \epsilon_{\text{joint}}^0 \leq J_{\text{st}}^c \qquad \forall \bar{t} \in \mathcal{T}, \tag{7.7}$$

$$0 \leq \sum_{t=1}^{\bar{t}} s_{\text{joint}}^t \Delta\tau + \sum_{n \in \mathcal{N}} \epsilon_{\text{joint},n}^0 \leq \sum_{n \in \mathcal{N}} J_{\text{st},n}^d \qquad \forall \bar{t} \in \mathcal{T}, \tag{7.8}$$

respectively. The collective battery storage variable is denoted by s_{joint}^t, and $\epsilon_{\text{joint}}^0$ or the $\epsilon_{\text{joint,n}}^0$ characterize the initial state of charge.

Power network model adaptations

The second-order cone formulation of the power flow equations, cf. (6.15), remains in force. However, the physical net load needs to be adjusted according to the considered configuration. In addition, there may be a centralized node hosting DERs.

If joint investment with decentralized DER installation is considered, the expressions of physical net loads are

$$p_n^t = \sigma_{\text{cast}}^t J_{\text{gen},n}^{\text{d}} + \kappa_n^t + \sum_{a \in \mathcal{A}_n} x_{n,a}^t + s_{\text{joint},n}^t \quad \forall n \in \mathcal{N}, \tag{7.9}$$

where $s_{\text{joint},n}^t$ is the power of the joint storage share physically located at node n.

If joint investment with centralized DER installation is considered, the expressions of physical net loads are

$$p_n^t = \kappa_n^t + \sum_{a \in \mathcal{A}_n} x_{n,a}^t \quad \forall n \in \mathcal{N}, \tag{7.10}$$

$$p_{cent}^t = \sigma_{\text{cast}}^t J_{\text{gen}}^{\text{c}} + s_{\text{joint}}^t, \tag{7.11}$$

where p_{cent}^t is the physical net load of the node hosting the centrally located generation and storage facilities.

Note that if the central node coincides with a member node, the corresponding node n is complemented with the second term of (7.11). Besides, there can be several central nodes. In that case, (7.11) is multiplied and indexed by the number of nodes.

If individual investment with decentralized installations is considered, the expressions of physical net loads are

$$p_n^t = \sigma_{\text{cast}}^t (I_{\text{gen},n,n}^{\text{d}} + \sum_{m \in \mathcal{N} \setminus n} I_{\text{gen},m,n}^{\text{d}}) + \kappa_n^t + \sum_{a \in \mathcal{A}_n} x_{n,a}^t + s_{n,\text{ind}}^t + s_{n,\text{host}}^t \quad \forall n \in \mathcal{N}. \tag{7.12}$$

If individual investment with centralized installations is considered , the expressions of physical net loads are

$$p_n^t = \kappa_n^t + \sum_{a \in \mathcal{A}_n} x_{n,a}^t, \tag{7.13}$$

$$p_{cent}^t = \sum_{n \in \mathcal{N}} \sigma_{\text{cast}}^t I_{\text{gen},n}^{\text{c}} + \sum_{n \in \mathcal{N}} (s_{n,\text{ind}}^t + s_{n,\text{host}}^t), \tag{7.14}$$

depending if the assets are decentralized or centrally installed.

Cost structure adaptations

The daily operational costs estimates are based on the same structure as the day-ahead energy exchange scheduling, cf. Chapter 6. They are:

- The *commodity costs.*
- The *upstream grid costs.*
- The *local grid costs.*

The initial investment module introduces new costs:

- The *investment costs:* they correspond to the cost of acquisition for the photovoltaic facilities and the storage systems. Each asset has a purchase price; γ_{gen} (in €/kWc) for the solar panels and γ_{st} (in €/kWh) for batteries. The investment cost is due at the initial time and it is obtained by

$$C_{\text{inv}} = \begin{cases} \gamma_{\text{gen}} J^c_{\text{gen}} + \gamma_{\text{st}} J^c_{\text{st}}, \\ \gamma_{\text{gen}} \sum_{n\in\mathcal{N}} J^d_{\text{gen},n} + \gamma_{\text{st}} \sum_{n\in\mathcal{N}} J^d_{\text{st},n}, \\ \gamma_{\text{gen}} \sum_{n\in\mathcal{N}} I^c_{\text{gen},n} + \gamma_{\text{st}} \sum_{n\in\mathcal{N}} I^c_{\text{st},n}, \\ \gamma_{\text{gen}} \sum_{n\in\mathcal{N}} I^d_{\text{gen},n,m} + \gamma_{\text{st}} \sum_{n\in\mathcal{N}} I^d_{\text{st},n,m}, \end{cases} \tag{7.15}$$

  depending if the investment is joint centralized, joint decentralized, individual centralized, and individual decentralized, respectively.

Remark 7.1. Battery storage systems may have two price components. In such a case, the power rating is treated as a variable that replaces $\mu^{\text{ch}} I_{\text{st},n}$ in (7.3). It has a corresponding price (in €/kW).

Typical days

Estimating daily expenses over a time span of several years or decades (the assets lifetime) would require massive computation efforts. Indeed, daily cost estimates variables are multiplied by the number of days. Besides, it is not efficient because many days present similar consumption and generation patterns. It is interesting to identify several representative days to be estimated [142], [143]. When datasets of past smart metering data are available, two main methods can be applied.

i **Manual segmentation**: electricity consumption is directly related to human activity. It tends to present strong seasonal patterns whose frequency and intensity vary according to the type of appliance. It is relatively easy to identify periods of overall higher or lower consumption. They are based on

   - *Meteorological causes*: seasons (colder or warmer time periods), sunny days, cloudy days, etc.
   - *Human causes (habits)*: weekdays, weekends, holidays, etc.

   The dataset can easily be segmented against one or several considerations. Groups of "similar" days are thus formed, and one or more days among each of them can be selected as typical days. It is also possible to define an average day among the groups. The methodology for photovoltaic production profiles is similar, but only meteorological causes are relevant to discriminate the related dataset.

ii **Clustering (machine learning)**: clustering is an automatic unsupervized technique. It aims at grouping objects (daily load vectors in this case) using statistical data analysis (e.g., using mathematical distances between objects). There may be clusters holding observable properties such as highlighted for segmentation, but it is not guaranteed. There are many different clustering algorithms such as k-mean, k-medoid, hierarchical clustering [144]–[146].

Opting for one technique or the other has advantages and drawbacks. The first ensures the interpretability of the clusters, but the second is usually less tedious and can exploit statistical information that is not easily identifiable.

A major handicap of typical days is its limited time frame which fails to reproduce transitions between days and processes taking place over several days. Instead of using this time frame, it is possible to opt for windows of several days.

For more representative behaviors, sequential modeling can better express transitions (especially between different days). Based on history, the model can be used to simulate the desired number of days. Popular sequential modeling techniques are time series, Markov chains, and machine learning. The occasional activities and behavior as well as the daily variability in the time of activities introduce strong stochasticity. It is a challenge to model such data at the individual level. Modeling consumption and generation at an individual level was the initial subject of the thesis. In particular, Seasonal Auto-Regressive Moving Average (SARMA) time series were used and several publications were made, cf. [147], [148].

7.2.4. Formulation

Similar to the scenarios developed in the previous chapters, the investment problem can be described by a game. Indeed, it is reasonable to assume that either self-interest will persuade the investment or that a directed investment policy can be legitimized only upon guaranteeing an equilibrium. However, two types of problems can be distinguished:

1. **Convex optimization problem.** Joint investment planning aims at minimizing the overall community cost. Although it can be formalized as a game, it may be studied and treated as a regular convex optimization problem. Therefore, the methodology is similar to that adopted in the previous chapters when daily proportional billing is adopted.

2. **Generalized Nash equilibrium problem.** The individual investment planning does not yield the social optimum as it is discussed in the following. In addition, because of the mutualization mechanisms and the power flow constraints, which all introduce shared constraints, it leads to a generalized Nash equilibrium problem. As exposed in 3.3.2, GNEPs are almost intractable. Instead, this specific type of GNEP with shared constraints can be reformulated as a regular NEP to compute the variational solution (all the other solutions are lost), which in addition features the interesting property that all the Lagrange multipliers associated with the global constraints are equal among players, cf. 3.3.3. Hence, the solution for which each individual's strategy contributes equally to each shared constraint is enforced.

Remark 7.2. The daily costs estimation model is treated as a single optimization problem so as to incorporate it smoothly in the global investment problem. In chapter 5, reducing the game to such a framework was made possible by adopting either a net load proportional billing or marginal cost pricing. However, as examined in that same chapter, adopting a continuous proportional billing leads to very limited inefficiency (cf. 5.3.3). The adoption of the latter billing for the day-ahead scheduling would therefore not change the investment results substantially.

Joint investment

Under such a configuration, the cost function to be minimized is

$$f(\Lambda) = \gamma_{\text{gen}} J_{\text{gen}} + \gamma_{\text{st}} J_{\text{st}} + \sum_{d \in \mathcal{D}} w_{\text{TD},d}\, g(\Lambda_{\text{TD},i}), \tag{7.16}$$

where $w_{\text{TD},d}$ is the weight attributed to typical day d, and

$$\begin{aligned} g(\Lambda_{\text{TD}}) = \sum_{t \in \mathcal{T}} \Delta\tau \Bigg[& \sum_{i \in \mathcal{S}} \left(\gamma^t_{\text{com},i} L^{t+}_i\right) + \gamma_{\text{grid}} \left(L^t \Delta\tau\right)^2 \\ & + \sum_{(j,k) \in \mathcal{B}} \left(\gamma_{\text{loss}} \left(p^t_{j,k} + p^t_{k,j}\right) + \gamma_{\text{flow}} \left(p^{t+}_{j,k} + p^{t-}_{j,k}\right) \delta_{j,k}\right) \Bigg]. \end{aligned} \tag{7.17}$$

Again, J_{gen} and J_{st} are the total investment capacities in DERs.

The joint investment problem is therefore solved by:

$$\begin{aligned} & \min_{\Lambda} f(\Lambda) \text{ as in (7.16)} \\ & \text{s.t. (7.4)–(7.11), (6.15).} \end{aligned} \tag{7.18}$$

Investment components add linear terms. Hence, the optimization problem remains convex. There is no particular difficulty in comparison to problems solved in chapters 4 to 6 under a daily proportional billing. However, the number of variables is significantly higher and depends on the number of selected typical days.

Individual investment

In this setting, the cost function to be minimized by each individual n is

$$f_n(\Lambda_n) = \gamma_{\text{gen}} \sum_{n\in\mathcal{N}} I_{\text{gen},n} + \gamma_{\text{st}} \sum_{n\in\mathcal{N}} I_{\text{st},n} + \sum_{i\in\mathcal{D}} o_{\text{TD},n,i}\, g(\Lambda_{\text{TD},n,i}), \tag{7.19}$$

where

$$g(\Lambda_{\text{TD},n}) \sum_{t\in\mathcal{T}} \left(\gamma^t_{\text{com},i} l^{t+}_n \Delta\tau + \gamma_{\text{grid}} l^t_n L^t (\Delta\tau)^2\right). \tag{7.20}$$

The individual investment problem consists therefore of N optimization problems

$$\begin{array}{ll} \min\limits_{\Lambda_n} & f_n(\Lambda_n) \\ \text{s.t.} & (7.1)-(7.3), (7.12)-(7.14), (6.15), (6.12). \end{array} \quad \forall n \in \mathcal{N}. \tag{7.21}$$

which are coupled by the shared constraints. This leads to a generalized Nash equilibrium problem as discussed earlier. A suitable variational inequality can be solved to obtain the variational solution.

Remark 7.3. Investment problems should probably be enhanced with additional constraints. Physical considerations imply limitations. For instance, congestions should be avoided by defining a maximum PV capacity on the network.

7.2.5. Investment participation cost

Joint investment

Joint investment raises the same questions as in the scenarios of day-ahead scheduling using daily proportional billing. Many different criteria can be used to distribute the investment cost. One way is to consider the net positive load. Members would pay an initial amount based on their consumption history. Then, at regular intervals, adjustments would be made based on the actual consumption. Another one is to consider the relative gains obtained throughout time.

Individual investment

Concerning individual investment, participation is straightforward. The individual pays the entire individual investment. However, there is one case that deserves attention. When a member n invests in assets that are physically located in member m's property, i.e., $I^d_{\text{gen},n,m}$ and/or $I^d_{\text{st},n,m}$. In line with objective 2, there is no reason to monetize the hosting of storage or generation capacity. It would introduce subjectivity, and the overall optimization would lose much of its meaning. Still, although it is subjective, it has a price for hosts (aesthetic impact, loss of space, increased hazard, etc.), and it seems reasonable to consider a reward mechanism. A simple mechanism could be the definition of an available share for the host. In such a case, the problem formulation should be adapted to account for this operational consideration. One will note that an individual will prioritize investing in its own capacities, but it may happen that some individuals cannot or do not wish to pay for such a considerable investment at once. Besides, such an agreement can benefit both parties when they have complementarities in terms of consumption. Then, it could be considered as a mutual investment, and it moves towards the joint investment framework.

7.3. Intraday settlement

7.3.1. Introduction

The scheduling solutions computed by the day-ahead schemes in chapters 4 to 6 are impossible to fully achieve. Reasons that make deviations unavoidable are of two natures.

i **Exogenous (natural) factors.** They are mostly due to meteorological conditions, which are impossible to forecast with full accuracy. Depending on the weather, photovoltaic generation produces more or less power, sometimes with fast and frequent intensity changes. Heating and cooling systems also react more or less to keep a constant temperature inside homes. Examples of this nature are numerous.

ii **Endogenous (human) factors.** For many reasons, end-users may change their plans and impact their consumption because of some event or just subjectively. Their schedule can thus be changed, and some loads may be added, canceled, or differed.

Note that in reality, many deviations are due to a combination of the two factors. For instance, if it gets dark during the day because of heavy cloud conditions, one may turn the light on depending on its perception.

The consequences of energy scheduling deviations can be significant. These consequences are translated by the cost denominator: the bill can decrease, but more probably, it will increase. For the same level of energy consumption, deviations from schedule imply a deviation from the optimum. In particular, the socialized component of the bill, i.e., at least grid costs, at most the whole bill, impacts the entire community. Hence, the question of whether the community as a whole or the individual concerned should incur the additional cost is raised. It is a difficult question with no true or right answer because life is full of unplanned events and exogenous factors are not controllable. It is essentially a social compromise. However, in the following, a few directions are proposed.

Remark 7.4. To prevent large additional intraday costs, it may be interesting to consider a robust day-ahead scheduling (cf. perspectives in Chapter 8). It would be damageable for the whole framework if just a small deviation would induce a major surcharge.

7.3.2. A few pointers

Ideally, actual energy exchanges should be settled following the billing defined in the day-ahead scheduling process. This is the only solution that would not change the analyses of the underlying games. Indeed, by modifying or adding rules on the intraday, the overall game is changed and may lead to different solutions. Practically, end-users could strategically select a position on the day-ahead horizon and deliberately deviate on the intraday horizon.

However, there are numerous options for intraday settlement that deserve each thorough developments. Some options are outlined as follows.

i **Status quo.** The community members are billed according to the initially defined billing (in day-ahead). All previous considerations remain valid. Given the systematic Nash equilibrium underlying all the developed billing schemes, no member has an interest in deviating from the agreed scheduling. The major drawback is that a careless member could penalize the community considerably without incurring much prejudice.

ii **Penalty charges.** The defaulting party is penalized by the payment of damages. It would be recommended to foresee an exemption quantity, i.e., a tolerance towards small and inevitable deviations. Note that the billing remains unchanged. However, the collected damages should probably be distributed to the rest of the community.

iii **Dedicated cost distribution of the extra cost.** The additional social costs are assessed and passed on to the accountable members with a dedicated distribution key. Some examples are:

- *Energy proportion.* The share of the total energy deviations directly defines the proportion of the costs incurred by the accountable member.
- *Marginal cost.* It is based on how much the extra cost is accountable to one member by resorting to the Shapley value or VCG.
- *Arbitrary rule.*

As previously highlighted, such distributions add a time horizon to the game. Special attention should be paid to prevent strategic plays involving the settlement process as deviations could be incentivized.

7.4. Conclusions

This chapter has given some extended perspectives for enriching the responsible energy community concept. The community is an entity that can cover all time horizons. Addressing all of them thus provides a significant advantage to raise maximum engagement and participation.

Hence, different formulations of DER investments accounting for four different configurations were presented. They were distinguished on the investment modality (individual or collective) and the physical installation of DERs (centralized or decentralized). In this way, many different situations can be addressed. Besides, to improve the investment planning, an estimation of operational costs based on the day-ahead scheduling scenarios was also included.

Besides, an insight on intraday settlement has been provided. Indeed, deviations and the possible strategies can significantly handicap the goal and the efficiency of the community.

It should be mentioned that both investment planning and intraday settlement deserve much more development. This is certainly the main perspective of this work.

CHAPTER 8.

Conclusions

This concluding chapter starts with an overview of the different frameworks and billings covered in chapter 4 to 6 and summarizes the main findings. Then, some perspectives are provided in Section 8.2. Finally, a series of general reflections are exposed in 8.3 before the final conclusions are drawn and close this manuscript.

8.1. Overview

Despite all the efforts to provide a leading thread and consistency to the manuscript, the work achieved in this thesis was in the main a non-linear process. It can thus rub off on the text and complicate the high-level analysis. Therefore, at this stage, it appears to be interesting to summarize. This section intends to provide a final overview that will help grasp the contributions that are further reminded and assessed.

8.1.1. On the energy exchange scheduling scenario

Chapters 4 to 6 introduced various day-ahead energy scheduling frameworks and used different computation methods. Table 4.1, discussed in 4.2.1 is reproduced here below to summarize the coverage of each chapter.

Hence, Chapter 4 assumed a basic scenario (without energy exchanges inside the community) without storage and mutualization that can be solved using regular convex optimization and the possible distributed algorithms. Chapter 5 assumed a basic scenario but included individual storage. Depending on the considered billing, either regular convex optimization or a solution based on variational inequality theory was used. Finally, Chapter 6 adopted more advanced frameworks based on energy exchanges and mutualization of excess resources. Billings resorted to convex

Table 8.1.: Summary of chapters' framework coverage in function of mathematical computation methods and the extent of the scenarios.

Involved chapters	Convex optimization	Variational inequalities
Basic scenario (ECS)	4 5 (storage included)	5
Mutualization (EES)	6	(6) perspective

optimization, but preliminary results of continuous proportional billing required a solution based on variational inequality.

8.1.2. On the community billing choice

Chapters 4 to 6 introduced a total of six different billings. While they present significant resemblance, they all have their characteristics. The billings are respectively:

1. **Inter-supplier billing [Inter],** cf. Chapter 4 (4.26).
2. **Net load proportional billing [Net],** cf. Chapter 5 (5.10).
3. **Marginal cost pricing [VCG],** cf. Chapter 5 (5.11).
4. **Continuous proportional billing [CP],** cf. Chapter 5 (5.12).
5. **Individual optimum proportional billing [Optimum],** cf. Chapter 6 (6.19).
6. **Shapley-based billing [Shapley],** cf. Chapter 6. (6.22)

The fairness of billings can only be assessed as part of the project that the community conveys, as discussed in 8.3.1. This work has provided different options that can meet different sensitivities towards that topic. Nonetheless, more objective considerations can be determined and compared:

- *Efficiency.* For the same flexibility level, it characterizes how close the aggregated cost is from the global (social) optimum.
- *Empowerment.* It describes how prone community members are to consent more or less flexibility.
- *Contract.* It distinguishes billings for which the supplying contract has an individual incidence (if not, it is socialized).

- *Network.* It identifies billings that pass on grid costs based on the actual use and separates it from flexibility performances.
- *Treatment.* It indicates if the billing tends to favor the advertised flexibility or the actual mobilized flexibility.

Table 8.2 summarizes the above considerations for each billing. Hence, there are no overall better billings. Instead, there are better billings regarding a defined aim. The details can be found in the respecting chapters. However, the main trends can be briefly outlined. All Strategy-proof scenarios lead to optimized social costs. Nash-incentive scenarios present some limited inefficiency, but it has to be weighed against the possible higher incentivization of flexibility. Continuous proportional billing is, in this way, particularly encouraging. The main difference lays in the philosophy of the billings. Three of them favor the pledged flexibility, whereas the other three favor the actual mobilized flexibility.

Table 8.2.: Summary of advantages (+), strong advantages (++), neutral concerns, the differentiation of network costs, and the type of rewarded flexibility.

	Efficiency	Empow.	Contract	Network	Treatment
Inter	+	neutral	yes	no	pledged
Net	+	neutral	no	no	pledged
VCG	+	+	yes	no	mobilized
CP	neutral	++	yes	yes	mobilized
Optimum	+	+	yes	no	pledged
Shapley	neutral	+	yes	yes	mobilized

8.2. Perspectives

Given the extent of the scope that communities cover, perspectives of research subjects are numerous. The more conceptual ones are:

- **Scheduling robustness.** It seems essential to address this issue when defining schedules. Indeed, the scenarios should ensure that small delays or other changes do not generate significant changes in the solution performance, i.e., limit the cost increase. Robust optimization [149], [150] and stochastic optimization [151], [152] formulations have proved to be relevant in such a setting. They implicitly provide some margin for the scheduling process (e.g., EV arrives too late for charging schedule).

- **Framework upscaling.** Another major line of research is to extend and scale up the proposed frameworks. Indeed, all communities use a common infrastructure, namely the medium-voltage and high-voltage grids. In addition, grid costs could be mutualized and distributed in a fair way to prevent local disparities in terms of infrastructure. In line with the willingness to provide the best reflection of the incurred costs, the sharing of the assets should be formalized. Interdependence would thus be materialized, and a game would take place between each community and other large end-users connected to the medium and high-voltage grids. The appropriate integration of communities within the electricity landscape is essential for the sector's integrity if they become widespread.

Generally, the contributions of this thesis address high-level and conceptual considerations. Many more practical and technical issues have to be raised before any community scenario can be implemented. Among them, there are:

- **Forecasting tools.** Non-dispatchable generation and non-flexible consumption (forming the baseload) have to be determined as accurately as possible to avoid significant intraday deviations. Defining models at such a microscopic level (one individual) with high fluctuations is challenging as previously exposed in 7.2.3. Time series modeling and artificial neural networks are the most relevant tools.

- **Designing costs and models.** All along with this work, simplified models were assumed for the technical constraints (e.g., lossless storage, fully flexible loads) and the costs (e.g., upstream grid costs). These models need to be refined to allow practical implementations.

- **Automation and control strategies.** The operating schedules of each device should be automated to facilitate the adoption by end-users. Besides, some flexible appliances could provide more flexibility options if the consumption could be modulated. Control strategies should also consider practical considerations such as the wear of some assets, e.g., manage battery charging and discharging cycles.

- **Algorithms design.** The algorithms used in this thesis are aimed at providing solutions. In practical implementations with large user groups and more elaborate models, the number of variables is multiplied. Particular attention should be paid to the choice of each mathematical method (e.g., optimization method) and the design of the algorithms. Moreover, necessary communication between members for distributed resolutions should be carefully assessed to ensure sufficient privacy.

- **Hardware definition.** The actual deployment of communities requires much equipment such as energy management systems, communication means, automation devices, etc. In parallel, compatibility issues, redundancy, resilience, durability, and many considerations also need to be addressed.

- **Regulatory framework.** The scenarios presented in this work imply significant adaptations of the regulations. Although some legal initiatives have started to shape a framework (e.g., European directives on energy communities, cf. Box 2.5), they are still incomplete and need further integration.
- **Sociological studies.** All the scenarios developed in this work are meaningless if end-users do not feel engaged or if they are simply reluctant to participate in a community. Besides, each billing generates different behaviors, which can also be dependent on the socio-economic context and other societal factors.

8.3. Concluding reflections

The treatment of such a topic, involving communities, has strong social implications. In this last section we shall reflect on some of these which are directly related to the objectives originally defined, cf. Section 1.2.

8.3.1. On fairness

Fairness is directly implied by sharing a common infrastructure. For example, each member should have fair access to the network and shared DERs. Sharing the Earth's resources and the externalities that energy exchange generate also implies fairness, but on a much larger scale.

Despite its ubiquity, there is no truth in fairness. Indeed, there is no single definition, and each of them is eminently political, in line with a societal project. Solidarity, however, has often been shown to enhance human potential and sustain life.

For this reason, this work proposed various billings with different approaches to fairness. Depending on the societal project (or ideology), one or the other billing is considered more appropriate.

8.3.2. On communities

The formation of communities has two immediate consequences: the discrimination and the gathering individuals and equipment. It is impossible to avoid either of these consequences. However, this proposal has attempted to discriminate primarily based on the basis of physically coherent grounds. For example, the perimeter of a community is tied to a particular electricity grid infrastructure. Moreover, the liberalized setting, which tends to individualize energy consumption by offering different pricing plans, is bypassed by several proposed billings.

More importantly, by recognising the sharing of electrical resources through aggregation, responsible energy communities naturally foster virtuous interactions that are further enhanced by the mutualization of excess resources.

8.3.3. On truth cost

Out of intellectual honesty, it should be said that there is no truth cost. There are only truth costs in relation to a project, a vision of what society should achieve. This proposition of responsible energy communities insist on a definition of true cost: one that minimizes environmental impact. However, it could be easily enhanced with a measure of comfort, workforce or even capital. The substance of the methods would remain unchanged because costs would still be non-linear.

Thus, there are many flaws, and perhaps this proposition is not practical or desirable for some, but the intent is at least to question past decisions and identify possible beliefs and dogmas. Hence, truth costs are closely related to fairness because of their societal dimension. The fair distribution of truth costs is again highly subjective.

8.3.4. On responsibility

The choices made for defining responsible energy communities commented on in the above reflections, aim to act "responsibly" with respect to a societal project. Perhaps it is naive, but it is intended to contribute to the maintenance of reasonable conditions for existing life.

Responsibility is thus a central feature that adds a concrete and consistent societal dimension to the very abstract concept of the energy community. There is a clear rupture with community-based markets, most of which advocate the fuzzy notion of market "efficiency".

8.4. Final conclusion

As mentioned several times in the manuscript, it is not the intention of this work to provide a turnkey tool for running day-ahead scheduling in a community. At best, the scenarios and mathematical framework could form the basis for more practical implementations. Instead, the work aims to highlight some features that a responsible model of energy communities could contain. Among the results, two key features have been developed.

Given the environmental challenge of this century, community-based scenarios should optimize the use of all natural resources. A cost design based on a close reflection of actual incurred costs is argued. Indeed, there should be as little room as possible for subjectivity. Electricity markets have multiplied enormously in recent decades. Based on simplistic models (efficiency assumptions are never met, cf. 2.5.1) for managing supply and demand, they have proliferated in the electricity sector as in many other former public services. Market imperfections and the interconnection of all electricity markets have led to a kind of chimera. Despite its mathematical appeal, designing new markets to govern exchanges within communities would likely complicate matters. A clear indicator, i.e., a meaningful definition of the price, should lead societal actions to successful enforcement of policies, especially environmental plans. Therefore, this proposal for responsible energy communities includes a bottom-up approach to cost based on physics. Of course, shared assets such as the power grid led to consider aggregated behaviors (e.g., aggregated power flows).

Secondly, optimization should lead to community scenarios and systems that release more potential for the same amount of mobilized resources. Mutualization and sharing have great potential in this regard. Moreover, they are useful and ubiquitous for living in society, but they have complicated implications for social relations. Instead, it has been increasingly supplanted by individualism. But again, beyond the political issue and the quote "sharing is caring", it is about the state of our planet. Therefore, this thesis has proposed several original scenarios mutualizing excess DER resources. It has increased the flexibility potential of communities, thus leading to lower costs.

On a personal note, I started my thesis with many certainties and admiration for the "beauty" of such an interesting service as electricity supply. I ended up questioning the whole frame. It is often said that the research world should not get involved in politics. I am of the opinion that we do have a political action. Engineers transform the planet. The energy sector even more so. The way we shape it is eminently political. Even if the main task of research is to develop new solutions, the question of societal benefits and the evaluation of the solutions in the light of human challenges should always be central. Scientists and engineers are most aware of the ins and outs. We need to warn, raise questions, identify, and, of course, to act.

Let's stop and think for a moment...

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